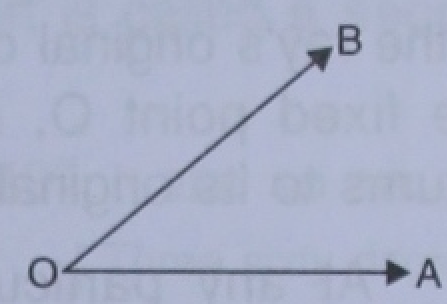
CHAPTER 17

ANGLES

(With their Types and Properties)

17.1 CONCEPT OF AN ANGLE

Two different rays starting from the same fixed point form an angle.



In the adjoining figure, two different rays OA and OB start from the same fixed point O to form angle AOB.

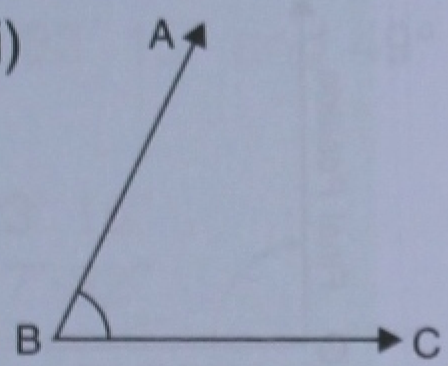
The point O, which is common to both the rays, is called the *vertex* of the angle AOB, whereas the rays OA and OB are called *the sides* or *arms* of the angle AOB.

The symbol '∠' is used to represent an angle.

Thus, angle AOB can be written as ∠AOB., *i.e.* angle AOB = ∠AOB.

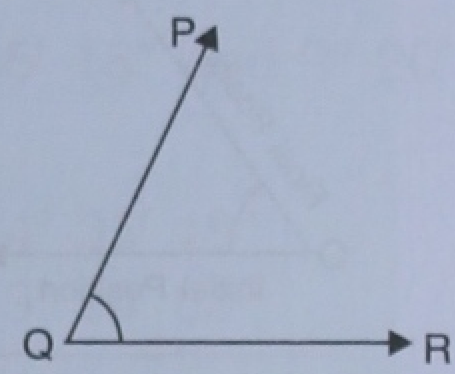
An angle is represented by three capital letters taken in such a way that the letter in the middle is always the vertex of the angle.

For example : (i)



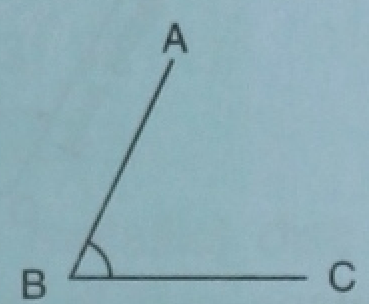
[∠ABC or ∠CBA]

(ii)

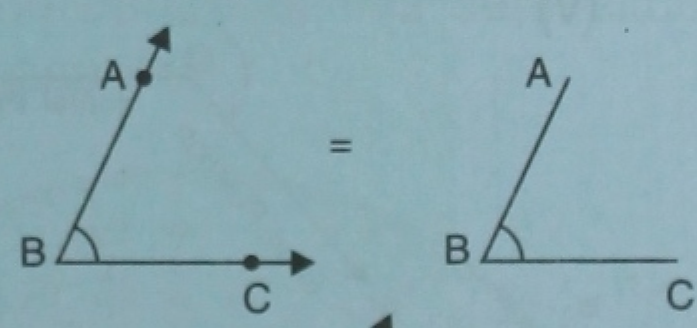


[∠PQR or ∠RQP]

1. An angle can be represented by the name of its vertex also.
   Thus, ∠ABC = ∠CBA = ∠B; ∠PQR = ∠RQP = ∠Q and so on.
2. Sometimes, two line segments with a common end point also form an angle at that point.

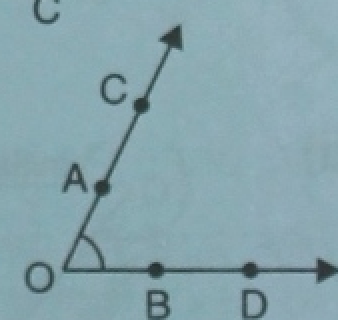
   

   In the adjoining figure, line segments BA and BC form angle ABC at their common end point.

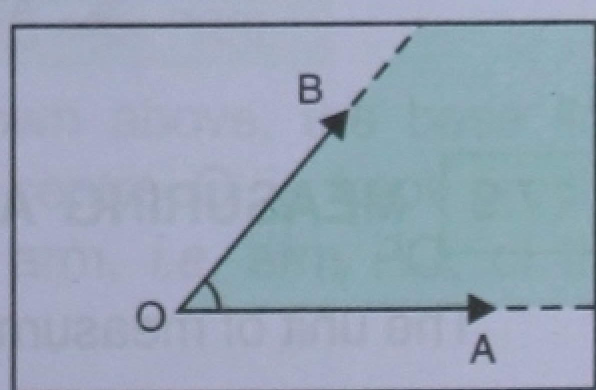
   It should be noted here that, whether or not BA and BC are rays or line segments, the measure of angle ABC remains the same.

   IMPORTANT : If the arms of an angle have several points on them, the same angle can be written in various ways.

   In the given figure : ∠O = ∠AOB = ∠BOA = ∠COB = ∠COD
   = ∠BOC = ∠DOA = ∠DOC = ∠AOD

17.2 INTERIOR OF AN ANGLE :



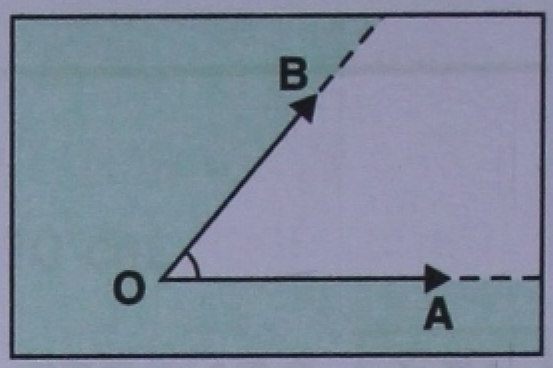
[Interior of an angle]

The interior of an angle is the region that lies within an angle. In other words, it is the region bounded by the arms of an angle.

The shaded portion of the given figure shows the interior of the angle AOB.

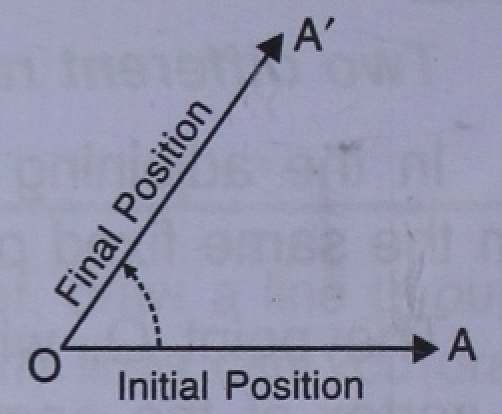
17.3 EXTERIOR OF AN ANGLE :

The exterior of an angle is the region that lies outside the angle. The shaded portion of the adjoining figure shows the exterior of the angle AOB.



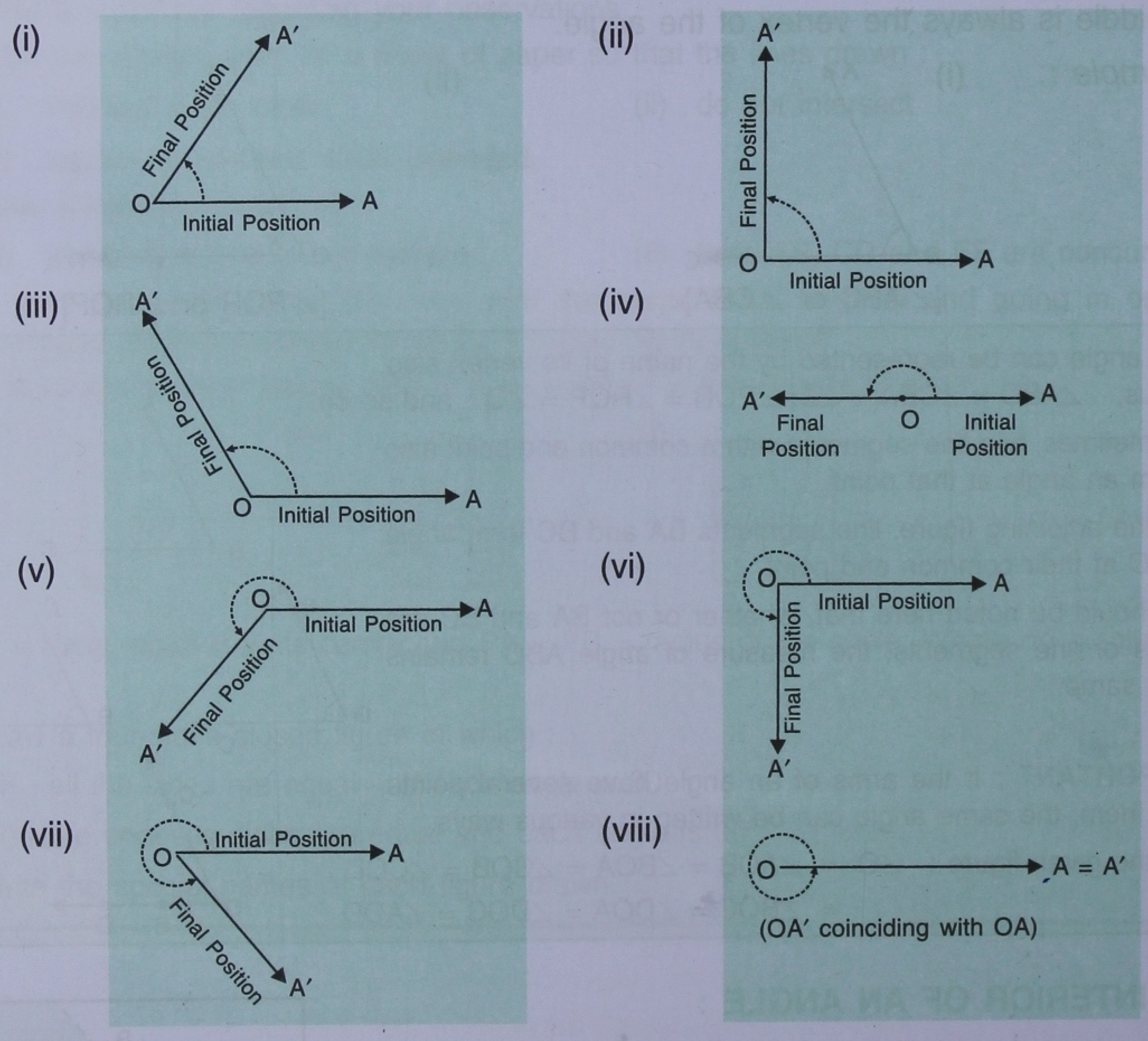
[Exterior of an angle]

17.4 ANGLE FORMED BY ROTATION :



Consider a ray OA rotating about its end point O (OA is the ray's original or initial position). As OA rotates about the fixed point O, it attains several positions before it returns to its original position.

At any particular moment, the position of ray OA, shown above as OA′, is termed its final position. It can be observed easily that the initial position OA and the final position OA′ form an angle ∠AOA′. The angle becomes bigger and bigger as OA′ rotates till it coincides with its initial position OA (see the following figures carefully).

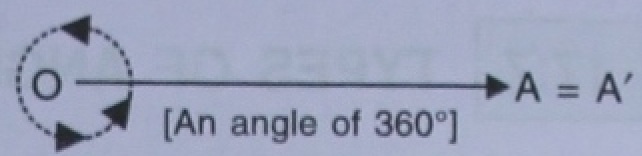


17.5 MEASURING AN ANGLE :

The unit of measurment of an angle is ***degree***. The symbol for degree is °.

Thus, if measure of an angle is **60 degrees**, we write **60°**.

The angle measure of one complete rotation is **360 degrees** *i.e.* **360°** [see alongside]



[An angle of 360°]

When the angle formed by one complete rotation of a ray is divided into 360 equal parts, each part is called one degree, *i.e.* 1°.

If one degree is further divided into 60 equal parts, each part is called a minute, which is denoted by **1′ (one prime)**.

Thus, 1 minute = 1′, 5 minutes = 5′, 40 minutes = 40′ and so on.

If we further divide one minute into 60 equal parts, each part is called **a second**, which is denoted by **1″ (two primes)**.

Thus, 1 second = 1″, 45 seconds = 45″, 40 seconds = 40″ and so on.

Hence, 1 complete rotation = 360° [Three sixty degrees]

1° = 60′ [Sixty minutes]

1′ = 60″ [Sixty seconds]

5 minutes 30 seconds = 5′ 30″

25 degrees 30 minutes 15 seconds = 25° 30′ 15″ and so on.

Example 1 :

Add : (i) 32° 23′ 15″ and 49° 17′ 32″ (ii) 74° 35′ 18″ and 9° 20′ 53″

Solution :

(i)
$$\begin{array}{r} 32^\circ\ 23'\ 15'' \\ +\ 49^\circ\ 17'\ 32'' \\ \hline 81^\circ\ 40'\ 47'' \\ \hline \end{array}$$
(Ans.)

(ii)
$$\begin{array}{r} 74^\circ\ 35'\ 18'' \\ +\ 9^\circ\ 20'\ 53'' \\ \hline 83^\circ\ 55'\ 71'' \\ \hline \end{array}$$

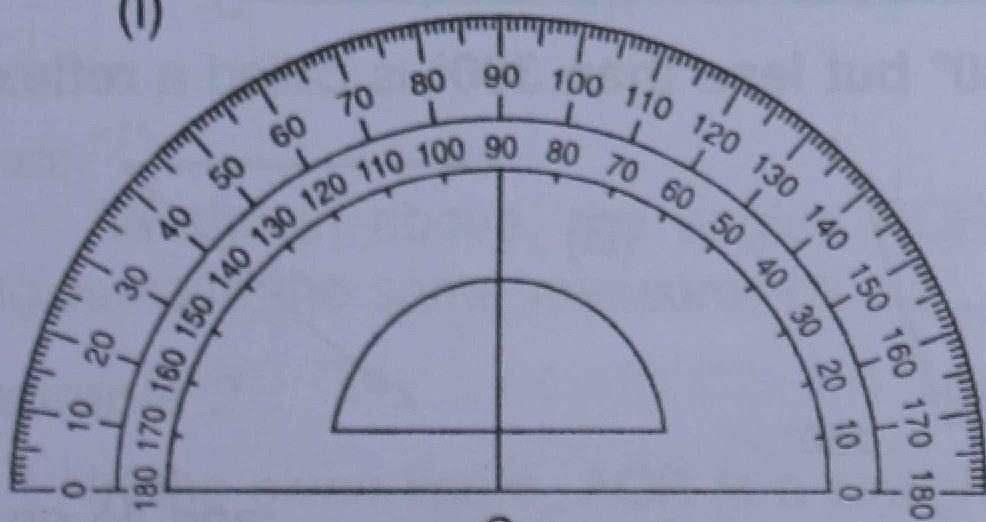
Since 60″ = 1′ ∴ 71″ = 1′ 11″

and so 83° 55′ 71″ = 83° 56′ 11″ **(Ans.)**

17.6 USING A PROTRACTOR FOR MEASURING AN ANGLE :

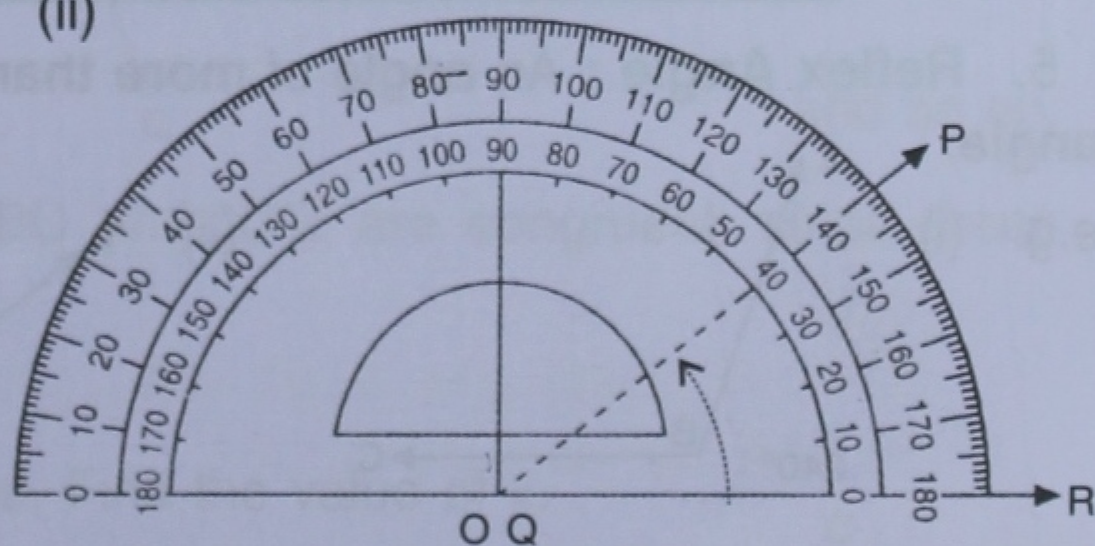
A protractor, as shown below, is a semi-circular plastic (or metallic) disc marked in degrees from 0° to 180° on its semi-circular part. The centre of this semi-circular piece is marked as O, which is also the mid-point of its base-line.

(i)



[A protractor]

(ii)



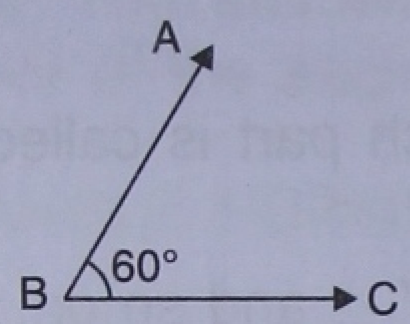
[Measuring angle PQR]

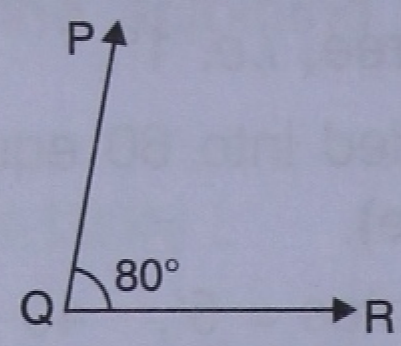
In order to measure an angle, say angle PQR, as shown above, the base line of the protractor is kept on arm QR of the angle, and its centre O is kept on the vertex of the angle the PQR. Now the position of the other arm, *i.e.* arm PQ, of the angle is read from the markings on the protractor.

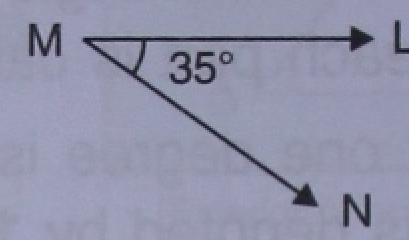
In the figure given above ∠PQR = 40°

17.7 TYPES OF ANGLES :

1. **Acute Angle :** An **angle** of **less than 90°** is known as an **acute angle.**

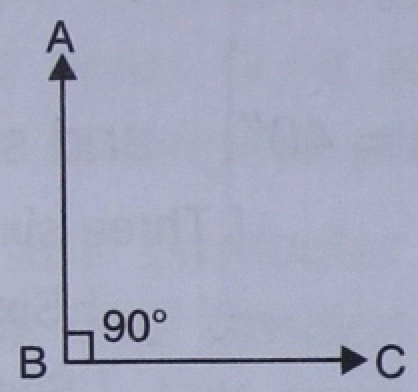
e.g. (i)


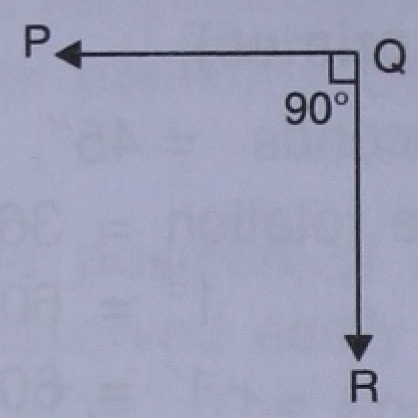
(ii)


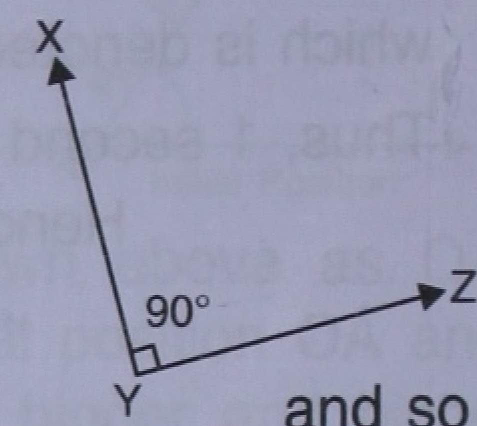
(iii)


and so on.

2. **Right Angle :** An **angle** of **90°** is known as a **right angle.**

e.g. (i)


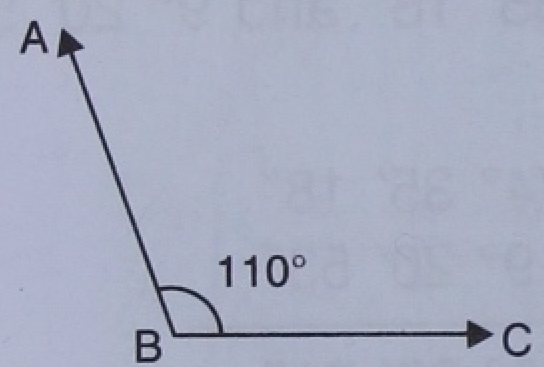
(ii)


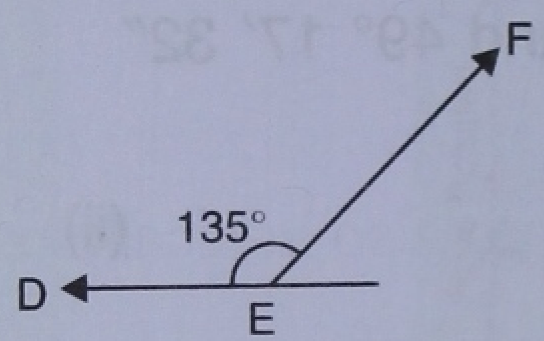
(iii)


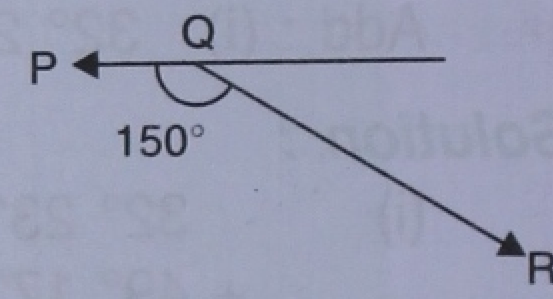
and so on.

Two lines (rays, line segments) that form an angle of 90° are said to be perpendicular to each other.

3. **Obtuse Angle :** An **angle** of **more than 90° but less than 180°** is known as an **obtuse angle.**

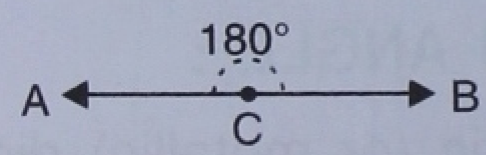
e.g. (i)


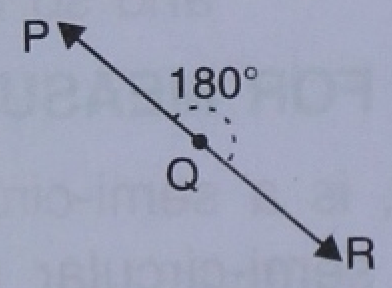
(ii)


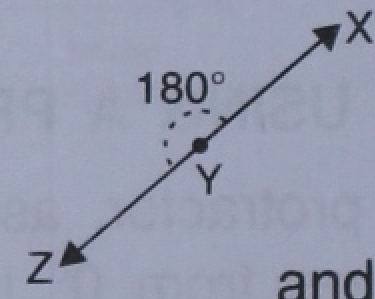
(iii)


and so on.

4. **Straight Line Angle :** An **angle** of **180°** is known as a **straight angle** or **straight line angle.**

e.g. (i)


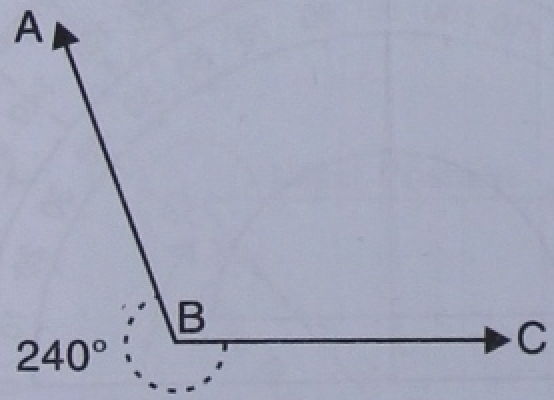
(ii)


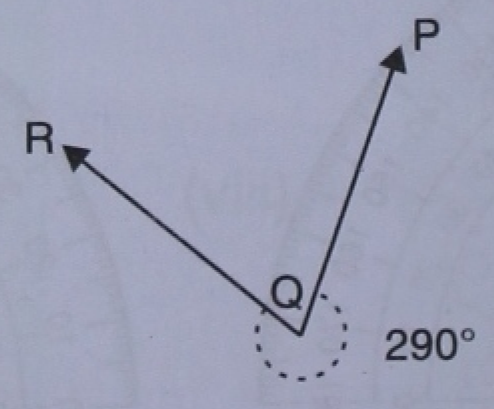
(iii)


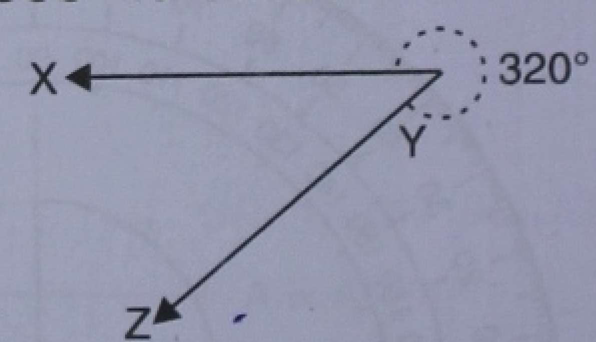
and so on.

A straight line angle = 180° = 2 × 90° = Two right angles

5. **Reflex Angle :** An **angle** of **more than 180° but less than 360°** is called a **reflex angle.**

e.g. (i)


(ii)


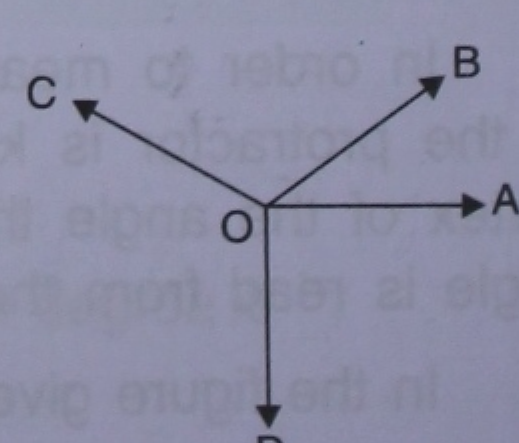
(iii)


and so on.

17.8 AN IMPORTANT RESULT :

The sum of the angles around a point is always 360°.

For example :



In the adjoining figure,

$\angle AOB + \angle BOC + \angle COD + \angle DOA = 360°$.

17.9 ADJACENT ANGLES :

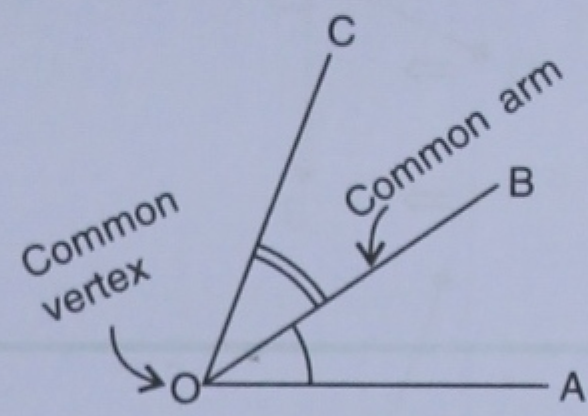
Two angles are said to be adjacent angles if :

(i) they have their vertices at the same point, *i.e.* they have a common vertex,

(ii) they have one common arm

and, (iii) the other arms of the angles are on the opposite sides of the common arm.

The figure given alongside shows two angles AOB and BOC having arm OB in common, vertex O in common, and also with their other arms OA and OC lying on opposite sides of the common arm OB.

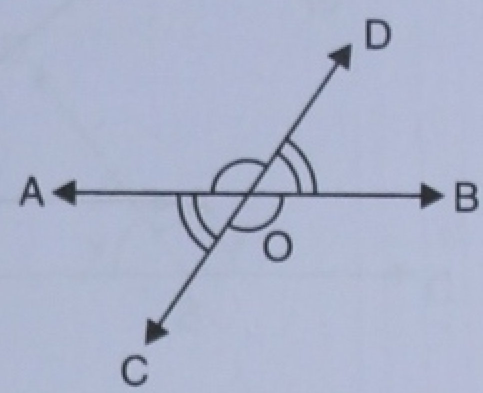


∴ ∠ AOB and ∠ BOC are adjacent angles.

17.10 VERTICALLY OPPOSITE ANGLES :

When two straight lines intersect, the angles on the opposite sides of their point of intersection are called vertically opposite angles.

The adjoining figure shows two lines AB and CD intersecting at point O.



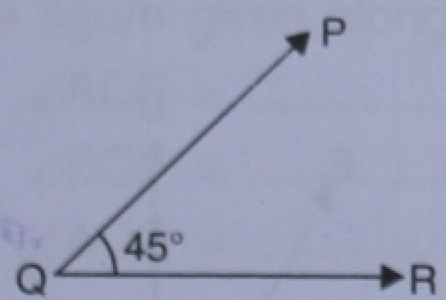
Clearly, ∠ AOC and ∠ BOD are vertically opposite angles and ∠ BOC and ∠ AOD are also vertically opposite angles.

> Vertically opposite angles are always equal.
> *i.e.* ∠ AOC = ∠ BOD and ∠ BOC = ∠ AOD

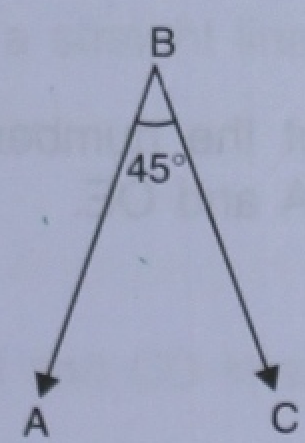
17.11 CONGRUENT (EQUAL) ANGLES :

Angles having the same measure (value), are said to be congruent angles.

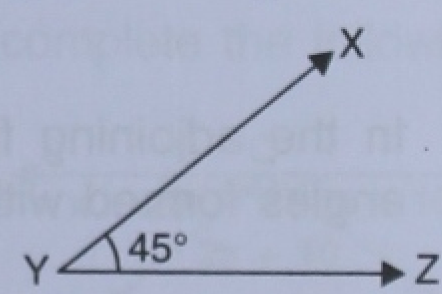
e.g. (i)



(ii)



(iii)

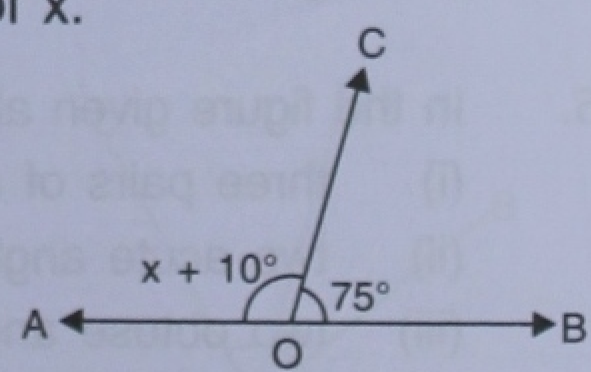


and so on.

As shown above, the angles PQR, ABC and XYZ are congruent, since these angles have the same measure, *i.e.* 45°.

Example 2 :

In the given figure, AOB is a straight line. Find the value of x.



Solution :

Since AOB is a straight line

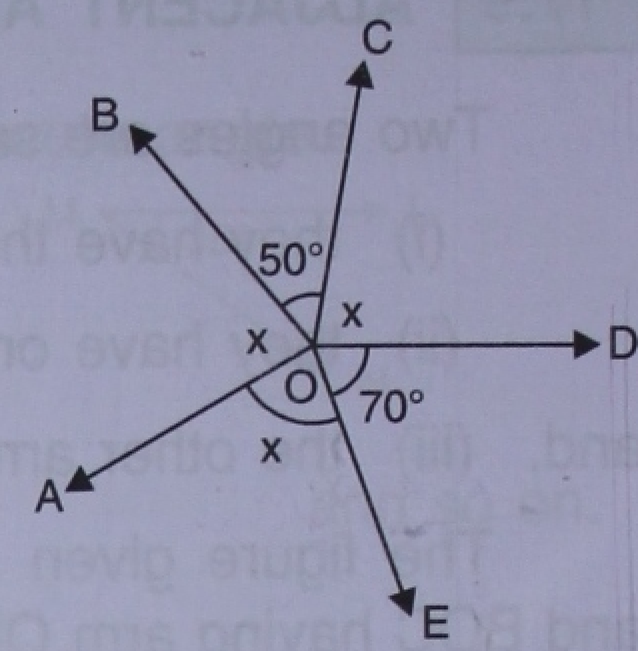
∴ $\angle AOB = 180° \Rightarrow x + 10° + 75° = 180°$

$\Rightarrow x + 85° = 180°$

$\Rightarrow$ **x** $= 180° - 85° =$ **95°** **(Ans.)**

Example 3 :

Use the given figure to find x.



Solution :

Since the sum of all the angles around a point is 360°.

$\therefore \quad x + x + 70^\circ + x + 50^\circ = 360^\circ$

$\Rightarrow \quad 3x + 120^\circ = 360^\circ$

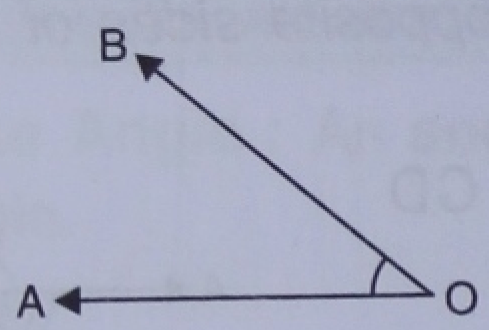
$\Rightarrow \quad 3x = 360^\circ - 120^\circ = 240^\circ$

$\Rightarrow \quad \mathbf{x} = \dfrac{240^\circ}{3} = \mathbf{80^\circ}$ **(Ans.)**

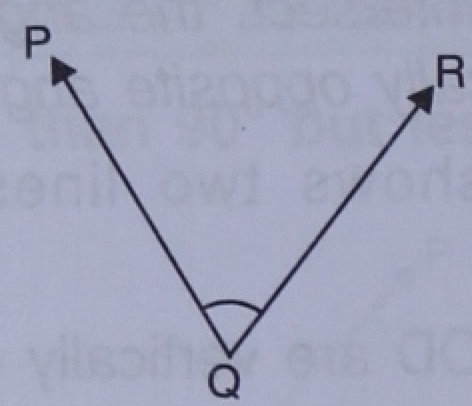
EXERCISE 17(A)

1. For each angle given below, write the name of the vertex, the names of the arms and the name of the angle.

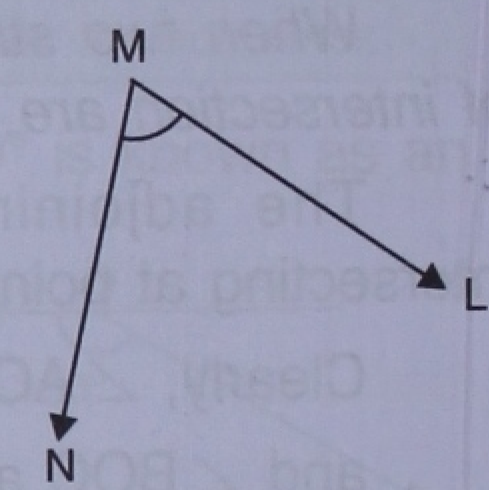
(i)



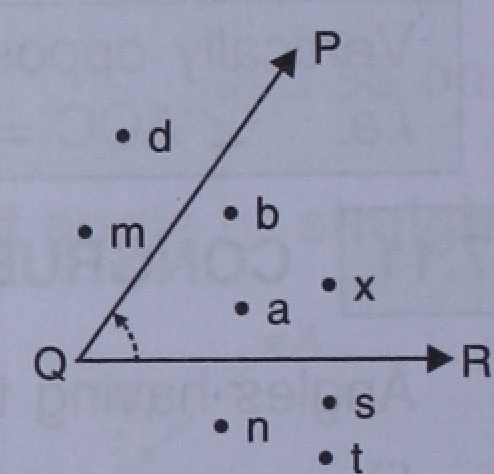
(ii)



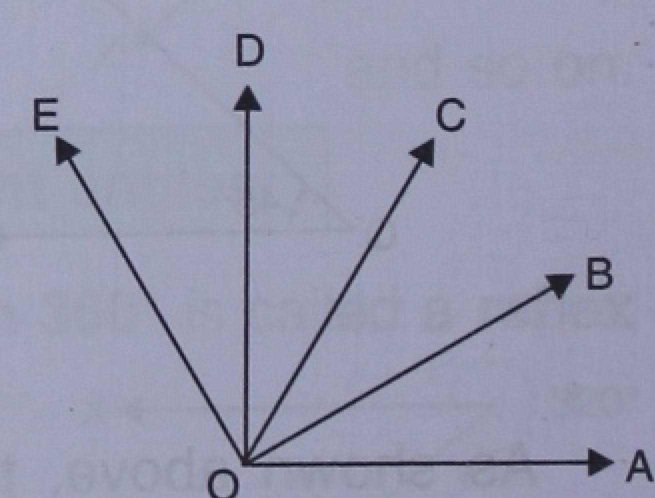
(iii)



2. Name the points :
   (i) in the interior of the angle PQR.
   (ii) in the exterior of the angle PQR.

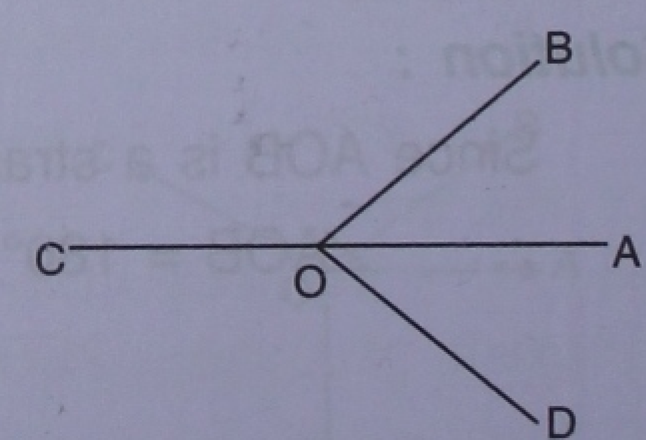


3. In the adjoining figure, figure out the number of angles formed within the arms OA and OE.



4. Add :
   (i) 29° 16′ 23″ and 8° 27′ 12″
   (ii) 9° 45′ 56″ and 73° 8′ 15″
   (iii) 56° 38′ and 27° 42′ 30″
   (iv) 47° and 61° 17′ 4″

5. In the figure given alongside, name :
   (i) three pairs of adjacent angles.
   (ii) two acute angles.
   (iii) two obtuse angles.
   (iv) two reflex angles.

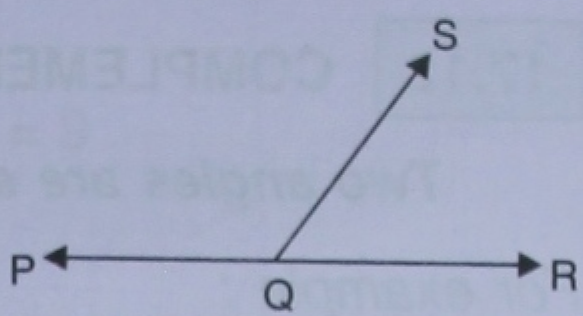


6. In the given figure :

PQR is a straight line. If :

(i) $\angle SQR = 75°$; find $\angle PQS$.

(ii) $\angle PQS = 110°$; find $\angle RQS$.

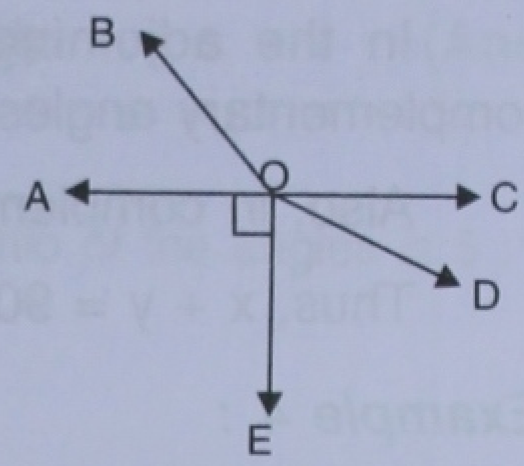


7. In the given figure, AOC is a straight line. If angle AOB = 50°, angle AOE = 90° and angle COD = 25°, find the measure of :

(i) angle BOC (ii) angle EOD

(iii) obtuse angle BOD (iv) reflex angle BOD

(v) reflex angle COE.

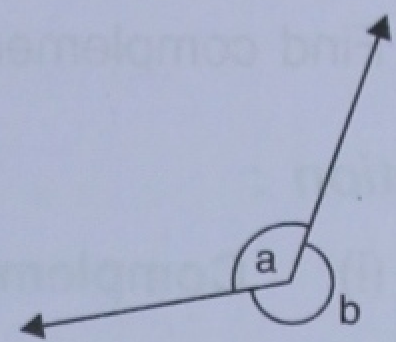


8. In the given figure, if :

(i) a = 130°, find b.

(ii) b = 200°, find a.

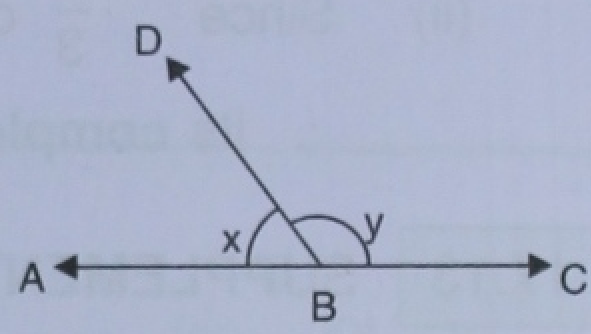
(iii) a = 5/3 right angle, find b.



9. In the figure given alongside, ABC is a straight line.

(i) If x = 53°, find y.

(ii) If $y = 1\frac{1}{2}$ right angles, find x.



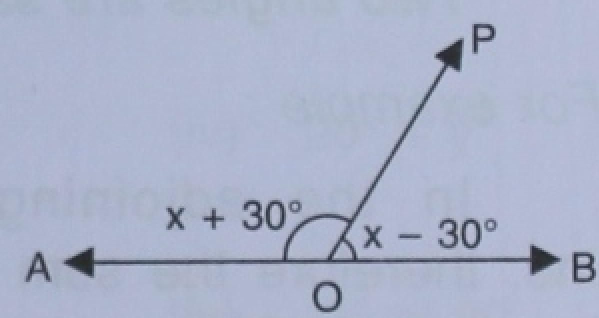
10. In the figure given alongside, AOB is a straight line. Find the value of x and also answer each of the following :

(i) $\angle AOP$ = ...

(ii) $\angle BOP$ = ...

(iii) which angle is obtuse ?

(iv) which angle is acute ?

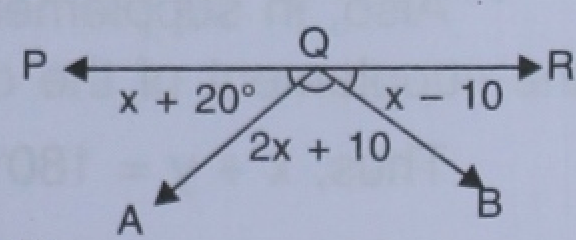


11. In the figure given alongside, PQR is a straight line. Find x. Then complete the following:

(i) $\angle AQB$ = ...

(ii) $\angle BQP$ = ...

(iii) $\angle AQR$ = ...



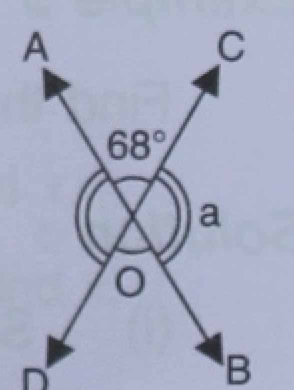
12. In the figure given alongside, lines AB and CD intersect at point O.

(i) Find the value of $\angle a$.

(ii) Name all the pairs of vertically opposite angles.

(iii) Name all the pairs of adjacent angles.

(iv) Name all the reflex angles formed and write the measure of each.

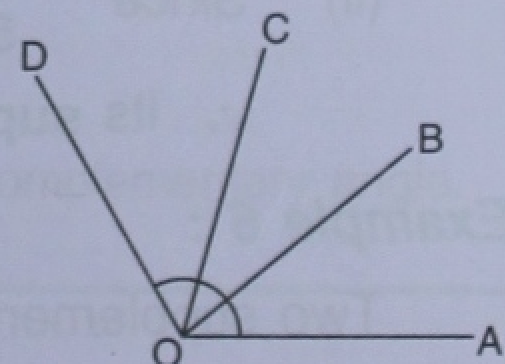


13. In the figure given alongside :

(i) if $\angle AOB = 45°$, $\angle BOC = 30°$ and $\angle AOD = 110°$; find angles COD and BOD.

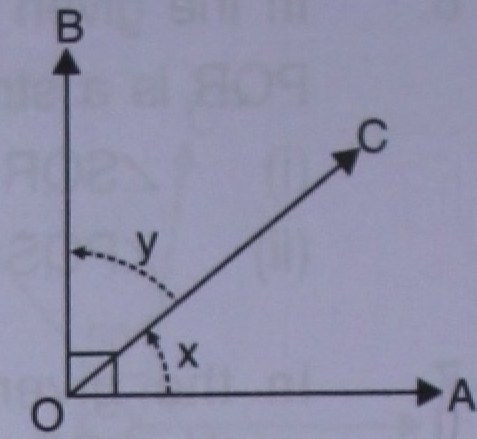
(ii) if $\angle BOC = \angle DOC = 34°$ and $\angle AOD = 120°$; find angle AOB and angle AOC.

(iii) if $\angle AOB = \angle BOC = \angle COD = 38°$; find reflex angle AOC and reflex angle AOD.



17.12 COMPLEMENTARY ANGLES :

Two angles are said to be complementary if their sum is 90°.



For example :

In the adjoining figure, angle x and angle y are complementary angles as their sum is 90°, *i.e.* x + y = 90°.

Also, in complementary angles; each angle is called the complement of the other.

Thus, x + y = 90° ⇒ (i) x is complement of y and (ii) y is complement of x.

Example 4 :

Find complement of each given angle : (i) 35° (ii) $\frac{2}{3}$ of 90°

Solution :

(i) **Complement of 35°** = 90° – 35° = **55°** **(Ans.)**

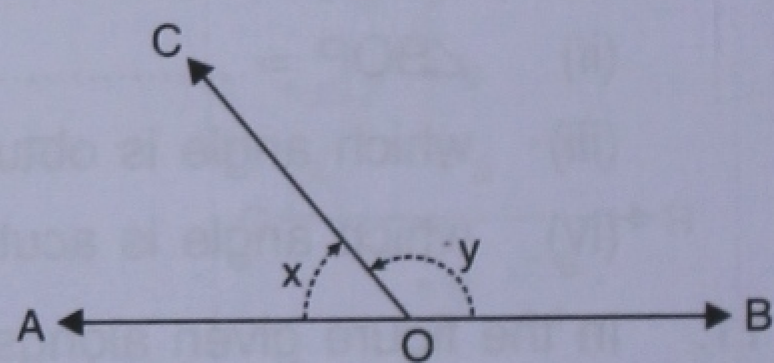
(ii) Since $\frac{2}{3}$ of 90° = $\frac{2}{3}$ × 90° = **60°**

∴ Its **complement** = 90° – 60° = **30°** **(Ans.)**

17.13 SUPPLEMENTARY ANGLES :

Two angles are said to be supplementary if their sum is 180°.

For example :



In the adjoining figure, AOB is a straight line; therefore the sum of the angles x and y is 180°, *i.e.* x + y = 180°.

Thus, angles x and y are supplementary angles.

Also, in supplementary angles, each angle is called the supplement of the other,

Thus, x + y = 180° ⇒ (i) x is the supplement of y and, (ii) y is the supplement of x.

Example 5 :

Find the supplement of each given angle : (i) 48° (ii) $\frac{4}{5}$ of 90°

Solution :

(i) **Supplement of 48°** = 180° – 48° = **132°** **(Ans.)**

(ii) Since $\frac{4}{5}$ of 90° = $\frac{4}{5}$ × 90° = **72°**

∴ Its **supplement** = 180° – 72° = **108°** **(Ans.)**

Example 6 :

Two supplementary angles are in the ratio 5 : 4. Find the angles.

Solution :

The ratio of the supplementary angles is 5 : 4, and 5 + 4 = 9

$\therefore$ The angles are $\frac{5}{9} \times 180°$ and $\frac{4}{9} \times 180°$

$= 100°$ and $80°$, respectively **(Ans.)**

Alternative method :

Let the angles be 5x and 4x. [As ratio of the angles is 5 : 4]

Since sum of supplementary angles = 180°

$\Rightarrow$ $5x + 4x = 180°$

$\Rightarrow$ $9x = 180°$ and $x = 20°$

$\therefore$ **Required angles** $= 5x$ and $4x$

$= 5 \times 20°$ and $4 \times 20°$

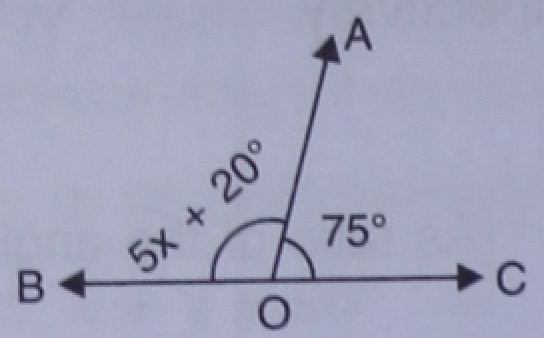
$= 100°$ and $80°$ **(Ans.)**

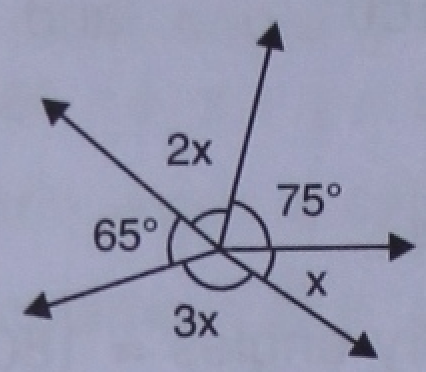
EXERCISE 17(B)

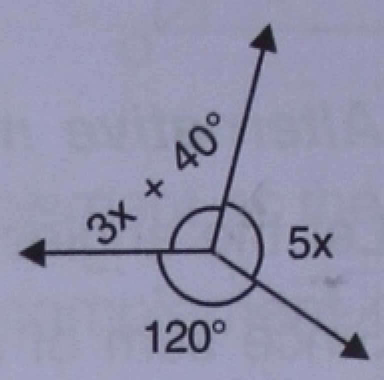
1. Write the complement angle of :
   (i) 45° (ii) x° (iii) (x – 10)° (iv) 20° + y°
2. Write the supplement angle of :
   (i) 49° (ii) 111° (iii) (x – 30)° (iv) 20° + y°
3. Find the complement angle of :
   (i) 1/2 of 60° (ii) 1/5 of 160° (iii) 2/5 of 70° (iv) 1/6 of 90°
4. Find the supplement angle of :
   (i) 50% of 120° (ii) 1/3 of 150° (iii) 60% of 100° (iv) 3/4 of 160°
5. Find the angle : (i) that is equal to its complement ?
   (ii) that is equal to its supplement ?
6. Two complementary angles are in the ratio 7 : 8. Find the angles.
7. Two supplementary angles are in the ratio 7 : 11. Find the angles.
8. The measures of two complementary angles are (2x – 7)° and (x + 4)°. Find x.
9. The measures of two supplementary angles are (3x + 15)° and (2x + 5)°. Find x.
10. For an angle x°, find :
    (i) the complementary angle
    (ii) the supplementary angle.
    (iii) the value of x° if its supplementary angle is three times its complementary angle.

Revision Exercise (Chapter 17)

1. Explain what do you understand by :
   (i) adjacent angles (ii) complementary angles (iii) supplementary angles

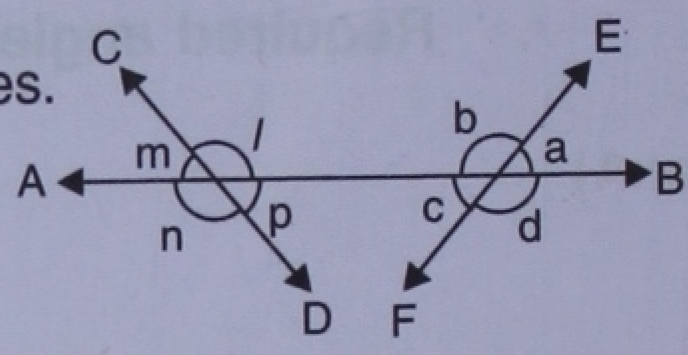
2. Find the value of 'x' for each of the following figures :
   (i) 

   (ii) 

   (iii) 

3. Find the number of degrees in an angle that is :
   (i) $\frac{3}{5}$ of a right angle (ii) 0·2 times of a straight line angle.

4. In the figure given alongside, AB, CD and EF are straight lines. Name the pair of angles forming :
   (i) straight line angles
   (ii) vertically opposite angles



5. Find the complement of : (i) $\frac{2}{5}$ of 210° (ii) 0.4 times of 130°

6. Find the supplement of : (i) $\frac{5}{7}$ of 154° (ii) 0.7 times of 150°

7. Two complementary angles are in the ratio 8 : 7. Find the angles.

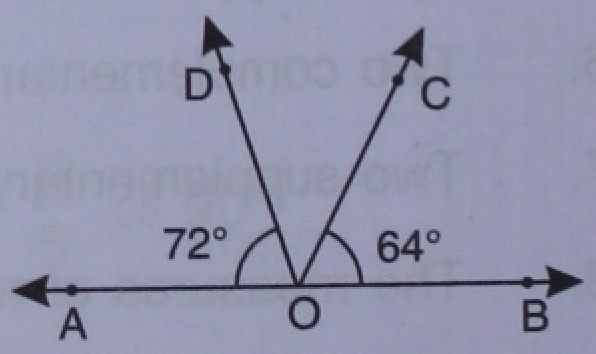
8. Two supplementary angles are in the ratio 7 : 5. Find the angles.

9. Two supplementary angles are $(5x - 82)°$ and $(4x + 73)°$. Find the value of x.

10. Find the angle formed by the arms of a clock at :
    (i) 3 O' clock (ii) 6 O' clock (iii) 9 O' clock (iv) 12 O' clock

11. For an angle y°, find :
    (i) its supplementary angle.
    (ii) its complementary angle.
    (iii) the value of y° if its supplement is four times its complement.

12. Use the adjoining figure to find :
    (i) ∠BOD
    (ii) ∠AOC



13. Two adjacent angles forming a linear pair are in the ratio 7 : 5; find the angles.

14. Find the angle that is three times its complementary angle.

15. An angle is one-thirds of a straight line angle; find :
    (i) the angle
    (ii) the complement and the supplement of the angle obtained above.