UNIT – 3 ALGEBRA

FUNDAMENTAL CONCEPTS

12.1 ALGEBRA

Algebra is a *generalized form of arithmetic.* In arithmetic, we use numbers like 5, –8, 0·64, etc., each with a definite value, whereas in algebra, we use letters (a, b, c, ..., x, y, z, etc.) along with numbers.

For example :

$7x$, $3x - 2$, $5a + b$, $2y - 5x$, $x + 2y - 7z$ and so on

The letters used in algebra are called ***variables*** or ***literal numbers*** or simply ***literals.*** They do not have a fixed value.

12.2 SIGNS AND SYMBOLS

In algebra, the signs +, –, × and ÷ are used in the same sense as they are used in arithmetic.

Also, the following *signs and symbols* are *frequently used in algebra,* each with the same meaning in every branch of mathematics.

$=$	means	*"is equal to"*	$\neq$	means	*"is not equal to"*
$<$	means	*"is less than"*	$>$	means	*"is greater than"*
$\nless$	means	*"is not less than"*	$\ngtr$	means	*"is not greater than"*
$\therefore$	means	*"therefore"*	$\because$	means	*"because"* or *"since"*
$\sim$	means	*"difference between"*	$\Rightarrow$	means	*"implies that".*

12.3 WRITING A GIVEN STATEMENT IN ALGEBRAIC FORM

Statement	Algebraic Form
(i) x subtracted from 8 is less than y	$8 - x < y$
(ii) y divided by 5 equals 2	$\frac{y}{5} = 2$
(iii) z increased by 2x is 23	$z + 2x = 23$

Conversely,

Algebraic Form	Statement
(i) $x + y = 3$	x plus y is equal to 3 or sum of x and y is equal to 3.
(ii) $p - 5 = x$	*p* minus 5 is equal to x or *p* decreased by 5 is equal to x. or *p* exceeds 5 by x
(iii) $5x > 7$	5 multiplied by x is greater than 7 or product of 5 and x is greater than 7
(iv) $\frac{8}{y} < 3$	8 divided by y is less than 3.

EXERCISE 12 (A)

1. Express each of the following statements in **algebraic form** :

(i)	The sum of 8 and x is equal to y.	....
(ii)	x decreased by 5 is equal to y.	....
(iii)	The sum of 2 and x is greater than y.	....
(iv)	The sum of x and y is less than 24.	....
(v)	15 multiplied by m gives 3n.	....
(vi)	Product of 8 and y is equal to 3x.	....
(vii)	30 divided by b is equal to p.	....
(viii)	z decreased by 3x is equal to y.	....
(ix)	12 times of x is equal to 5z.	....
(x)	12 times of x is greater than 5z.	....
(xi)	12 times of x is less than 5z.	....
(xii)	3z subtracted from 45 is equal to y.	....
(xiii)	8x divided by y is equal to 2z.	....
(xiv)	7y subtracted from 5x gives 8z.	....
(xv)	7y decreased by 5x gives 8z.	....

2. For each of the following algebraic expressions, write a suitable statement in words :

(i)	$3x + 8 = 15$	....
(ii)	$7 - y > x$	....
(iii)	$2y - x < 12$	....
(iv)	$5 \div z = 5$	....
(v)	$a + 2b > 18$	....
(vi)	$2x - 3y = 16$	....
(vii)	$3a - 4b > 14$	....
(viii)	$b + 7a < 21$	....
(ix)	$(16 + 2a) - x > 25$	....
(x)	$(3x + 12) - y < 3a$	....

12.4 CONSTANTS AND VARIABLES

In algebra, we come across *two types of symbols,* namely, *constants* and *variables.*

A symbol with a *fixed numerical value* in all situations is called a **constant**, e.g. 5, 30, 256, –7, $\frac{5}{3}$, $\frac{7}{9}$, etc., whereas a symbol whose *value changes with situation* is called a **variable,** such as; x, y, p, q, 5x, etc.

In 3x, 3 is a constant and x a variable but, together, 3x is a variable.
Reason : *As the value of x will change, the value of 3x will also change accordingly.*
Similarly 3 is constant and x is variable but, together, each of $3 + x$, $x - 3$ and $x \div 3$ is a variable.
So, we conclude that every combination of a constant and a variable is always a variable.

12.5 TERM

*A **term** is a constant or a variable or a product or a quotient of constants and variables.*

For example :

(i) 4 is a term; which is a *constant*
(ii) x is a term, which is a *variable*
(iii) 4x is a term; which is the *product of a constant and a variable.*
(iv) $\frac{3}{y}$ is a term; which is the *quotient of a constant and a variable.*

A term is called a **constant term** if it does not contain any literal (variable).

Thus, each of 3, –20, $\frac{5}{7}$, $-\frac{4}{9}$, etc. is a constant term.

Constants (fixed numbers) and variables (literal numbers) may be combined to form several types of terms.

For example :

The constants 2, 5, –8, 4, $\frac{3}{2}$, etc., and the variables x, y, z, etc., may be combined to form terms such as 2x, 5y, 5xy, 5xyz, 4xz, $\frac{3}{2}$yz, ...

(i) Like Terms :

The terms having *the same literal coefficients* are called ***like terms***. They may differ only in their numeral coefficients.

For example :

Each having the same literal coefficient : xy

(i) xy, 5xy, – 4xy, etc. are like terms
(ii) $-8x^2y$, $7x^2y$, $1{\cdot}5x^2y$, etc. are like terms and so on.

(ii) Unlike Terms :

The terms that *do not have the same literal coefficients* are called ***unlike terms***.

For example :

(i) 6a, 6ab and 6ac are unlike terms.
(ii) 2xy, $2x^2y$ and $2xy^2$ are unlike terms and so on.

12.6 ALGEBRAIC EXPRESSIONS

An algebraic expression is a collection of one or more terms, which are separated from each other by the signs + (plus) and/or – (minus).

For example :

Algebraic expressions	Number of terms used	Terms
(i) $5x$	1	$5x$
(ii) $8xy^2$	1	$8xy^2$
(iii) $3x + 8z$	2	$3x$ and $8z$
(iv) $4x - y + 7$	3	$4x$, y and 7
(v) $7xy + \frac{2a}{y} - 3z + 8$	4	$7xy$, $\frac{2a}{y}$, $3z$ and 8

and so on.

> In the algebraic expression $4x - y + 7$, 7 is the constant term as it does not contain a literal.
> Similarly, in the algebraic expression $7xy + \frac{2a}{y} - 3z + 8$; 8 is the constant term.

12.7 TYPES OF ALGEBRAIC EXPRESSIONS

According to the number of terms used to form an algebraic expression, it is called monomial, binomial, trinomial, and so on as explained below.

(i) Monomial :

An *algebraic expression* with *only one term* is called a ***monomial***.

For example : -8, z, xy, $2x$, $5y$, $\frac{2x}{5y}$, etc. are all monomials.

(ii) Binomial :

An algebraic expression of *two unlike terms* is called a ***binomial***.

A binomial is a polynomial of two terms

For example :

(i) $5x + 2y$, $7 - x$, $4x + y$, $y + zy$, etc. (ii) $2a + \frac{b}{2}$, $\frac{a}{3} - \frac{b}{3}$, $\frac{ab}{2} + \frac{26}{3}$, etc.

(iii) Trinomial :

An algebraic expression containing *three unlike terms* is called a ***trinomial***.

A trinomial is a polynomial of three terms

For example :

$ax^2 + bx + c$, $2x^2 - 7x + 4$, $xy - x + y^2$, etc.

(iv) Multinomial :

An algebraic expression with two or *more than two* terms is called a ***multinomial***.

For example :

(i) Each of $3x + 2$, $5 - x$, $a^2 - 7x$ is a *multinomial of two terms.*
(ii) $7 + x - xy + y^2$ is a *multinomial of four terms.*
(iii) $a + ab - b^2 + 7x - z$ is a *multinomial of five terms* and so on.

(v) Polynomial :

An *algebraic expression* with one *or more (unlike) terms,* is called a ***polynomial***.

For example :

(i) Each of –20, 8, x, 5x, $3xy^2$, etc., is a polynomial.

(ii) 3x + 2y is a *polynomial of two terms.*

(iii) x + 4yz – 7z + 8 is a *polynomial of four terms.*

(iv) Every monomial, every binomial, every trinomial and every multinomial is a polynomial.

(v) A polynomial can not be of the form : $\frac{1}{x}, \frac{3}{x+5}, \frac{2x}{x-5}, \frac{5}{x^2}, \frac{7x}{x^2+8}$, etc.

> *Terms are separated by plus (+) and minus (–) signs only.*
> *The signs of multiplication (×) and division (÷) do not separate terms.*
> *Thus, 3p + 5z – 7y has three terms, whereas 3p × 5z – 7y has two terms only.*
> *In the same way, 8 – 4x + 7y + 2z has four terms, whereas 8 × 4x × 7y ÷ 2z has only one term.*

12.8 PRODUCTS AND FACTORS

A **product** is the result of the multiplication of two or more constants or literals or of both.

For example :

5xy is the product of 5, x and y.

Each content and each literal multiplied together to form a product is called a **factor** of that product.

12.9 COEFFICIENT

Any factor or group of factors of a product is known as the **coefficient** of the remaining factors.

For example :

In the product 5axy,

5 is the coefficient of axy, 5x is the coefficient of ay, xy is the coefficient of 5a, axy is the coefficient of 5 and so on.

If a factor is a *numerical quantity*, it is called a *numeral coefficient* of the remaining factors, and if a factor involves *letters*, it is called a *literal coefficient* of the remaining factors.

For example :

In a product 3xy,

3 is a *numeral coefficient* of xy, x is a *literal coefficient* of 3y, xy is a *literal coefficient* of 3, y is *literal coefficient* of 3x, 3y is *literal coefficient* of x and so on.

> *When the coefficient is unity, i.e. 1 (one), it is usually omitted, i.e. 1b is written as b.*

12.10 POWER OF LITERAL QUANTITIES

When *a quantity is multiplied by itself* any number of times, the product is called *a power of that quantity*. This product is expressed by writing the number of like factors in it to the right of the quantity slightly raised.

For example :

$a \times a$ has 2 like factors, so to express it as : $a \times a = a^2$

Similarly, (i) $a \times a \times a$ has 3 like factors, so we write : $a \times a \times a = a^3$.

(ii) $a \times a \times a \times a \times a$ has 5 like factors, so we write : $a \times a \times a \times a \times a = a^5$.

The following table will make the concept, more clear :

Product	Write as :	Read as :
(i) $a \times a$	a^2	*a* squared or *a* raised to the power 2.
(ii) $a \times a \times a$	a^3	*a* cubed or *a* raised to the power 3.
(iii) $m \times m \times m \times m \times m$	m^5	*m* raised to the power 5 or fifth power of *m*.

In a^8, ***a*** is called the **base** and **8** is called the **exponent** or the **index** or the **power.**

Similarly, in x^5, **x** is the **base** and **5** is the **exponent** or the **index** or the **power** and so on.

1. For all values of x, $x^1 = x$ i.e. $5^1 = 5$, $8^1 = 8$, $35^1 = 35$	and so on
2. For all values of x, $x^0 = 1$ i.e. $5^0 = 1$, $8^0 = 1$, $35^0 = 1$	and so on

Example 1 :

Write each of the following products in **index form :**

(i) $m \times m \times n \times n \times n \times n$ (ii) $3 \times b \times b \times b \times b \times p \times p \times p$

Solution :

(i) $m \times m \times n \times n \times n \times n = \mathbf{m^2n^4}$ **(Ans.)**

(ii) $3 \times b \times b \times b \times b \times p \times p \times p = \mathbf{3b^4p^3}$ **(Ans.)**

Example 2 :

Write each of the following in **product form :**

(i) $3p^4$ (ii) $7b^2q^3$ (iii) $a^3m^4n^2$

Solution :

(i) $3p^4 = \mathbf{3 \times p \times p \times p \times p}$ **(Ans.)**

(ii) $7b^2q^3 = \mathbf{7 \times b \times b \times q \times q \times q}$ **(Ans.)**

(iii) $a^3m^4n^2 = \mathbf{a \times a \times a \times m \times m \times m \times m \times n \times n}$ **(Ans.)**

12.11 POLYNOMIAL IN ONE VARIABLE AND ITS DEGREE

When an **algeraic expression** is **made of one variable** only, it is called a **polynomial in one variable.**

For example :

(i) $3 + 5x - 7x^2$ is a polynomial in variable x.

(ii) $9y^3 - 5y^2 + 8$ is a polynomial in variable y.

The degree of a polynomial in one variable is the greatest of the exponents (powers) of its various terms.

For example :

1. For polynomial $4x^2 - 3x^5 + 8x^6$
   (i) the exponent of the term $4x^2 = 2$,
   (ii) the exponent of the term $3x^5 = 5$ and
   (iii) the exponent of the term $8x^6 = 6$.
   Since the greatest exponent is 6
   $\therefore$ The **degree** of the polynomial $4x^2 - 3x^5 + 8x^6 = \mathbf{6}$

The polynomial $3x^4 - x^3 + 5x - 7$ is in one variable only, the variable being x.
The polynomial $8y^5 - 3y^2 + 8$ is also in one variable only, the variable being y.

2. The **degree** of the polynomial $25 - x^4$ is **4.**
3. The **degree** of the polynomial $5x - 3$ is **1.**
4. The **degree** of the polynomial $4x^3 - 15x^5 - 7x^8$ is **8** and so on.

$\because x = x^1$

Polynomials of two or more variables and their degree

For example :

(i) $3x + xy^2 - 8yz$ is a polynomial made of three variables, x, y and z.
(ii) $5y^3 - 3y^2x + 8x^2y^2 - 3x^5$ is a polynomial of two variables, x and y.

In order to find the degrees of such polynomials, find :

(a) The sum of the powers of all the variables used in each term of a given polynomial.
(b) The greatest of these sum is the degree of the given polynomial.

For example :

(i) **For polynomial $3x + xy^2 - 8yz$**
The terms used are $3x$, xy^2 and $8yz$
Since the sum of the powers of the variables in $3x$ used = 1, $[3x = 3x^1]$
the sum of the powers of the variables in $xy^2 = 1 + 2 = 3$
and the sum of the powers of the variables used in $8yz = 1 + 1 = 2$
Clearly, **degree of the given polynomial = 3**

(ii) **In $5y^3 - 3y^2x + 8x^2y^2 - 3x^5$**
The sum of the powers of the term $5y^3 = 3$
the sum of the powers of the term $3y^2x = 2 + 1 = 3$
the sum of the powers of the term $8x^2y^2 = 2 + 2 = 4$
and the sum of the powers of the term $3x^5 = 5$
$\therefore$ **The degree of the given polynomial = 5**

EXERCISE 12 (B)

1. Separate the **constants** and the **variables** from each of the following :

$$6,\ 4y,\ -3x,\ \frac{5}{4},\ \frac{4}{5}xy,\ az,\ 7p,\ 0,\ \frac{9x}{y},\ \frac{3}{4x},\ -\frac{xz}{3y}$$

2. Group the like terms together :

(i) $4x, -3y, -x, \frac{2}{3}x, \frac{4}{5}y$ and y.
(ii) $\frac{2}{3}xy, -4yx, 2yz, \frac{-2}{3}yz, \frac{zy}{3}$ and yx.
(iii) $-ab^2, b^2a^2, 7b^2a, -3a^2b^2$ and $2ab^2$
(iv) $5ax, -5by, \frac{by}{7}, 7xa$ and $\frac{2ax}{3}$.

3. State whether **true** or **false** :

(i) 16 is a constant and y is a variable, but 16y is variable.
(ii) 5x has two terms 5 and x.
(iii) The expression $5 + x$ has two terms 5 and x.
(iv) The expression $2x^2 + x$ is a trinomial.
(v) $ax^2 + bx + c$ is a trinomial.
(vi) $8 \times ab$ is a binomial.
(vii) $8 + ab$ is a binomial.
(viii) $x^3 - 5xy + 6x + 7$ is a polynomial.
(ix) $x^3 - 5xy + 6x + 7$ is a multinomial.
(x) The coefficient of x in 5x is 5x.
(xi) The coefficient of ab in $-ab$ is -1.
(xii) The coefficient of y in $-3xy$ is -3.

4. State the number of terms in each of the following expressions :

(i) $2a - b$
(ii) $3 \times x + \frac{a}{2}$
(iii) $3x - \frac{x}{p}$
(iv) $a \div x \times b + c$
(v) $3x \div 2 + y + 4$
(vi) $xy \div 2$
(vii) $x + y \div a$
(viii) $2x + y + 8 \div y$
(ix) $2 \times a + 3 \div b + 4$

5. State whether **true** or **false** :

(i) xy and $-yx$ are like terms.
(ii) x^2y and $-y^2x$ are like terms.
(iii) a and $-a$ are like terms.
(iv) $-ba$ and $2ab$ are unlike terms.
(v) 5 and 5x are like terms.
(vi) 3xy and 4xyz are unlike terms.

6. For each expression given below, state whether it is a *monomial, or a binomial* or a *trinomial.*

(i) xy
(ii) $xy + x$
(iii) $2x \div y$
(iv) $-a$
(v) $ax^2 - x + 5$
(vi) $-3bc + d$
(vii) $1 + x + y$
(viii) $1 + x \div y$
(ix) $x + xy - y^2$

7. Write down the coefficient of x in the following monomials :

(i) x
(ii) $-x$
(iii) $-3x$
(iv) $-5ax$
(v) $\frac{3}{2}xy$
(vi) $\frac{ax}{y}$

8. Write the coefficients of :

(i) x in $-3xy^2$
(ii) x in $-ax$
(iii) y in $-y$
(iv) y in $\frac{2}{a}y$
(v) xy in $-2xyz$
(vi) ax in $-axy^2$
(vii) x^2y in $-3ax^2y$
(viii) xy^2 in $5axy^2$

9. State the numeral coefficients of the following monomials :

(i) $5xy$
(ii) abc
(iii) $5pqr$
(iv) $\frac{-2x}{y}$
(v) $\frac{2}{3}xy^2$
(vi) $\frac{-15xy}{2z}$
(vii) $-7x \div y$
(viii) $-3x \div (2y)$

10. Write the degree of each of the following polynomials :

(i) $x + x^2$
(ii) $5x^2 - 7x + 2$
(iii) $x^3 - x^8 + x^{10}$
(iv) $1 - 100x^{20}$
(v) $4 + 4x - 4x^3$
(vi) $8x^2y - 3y^2 + x^2y^5$
(vii) $8z^3 - 8y^2z^3 + 7yz^5$
(viii) $4y^2 - 3x^3 + y^2x^7$

REVISION EXERCISE (Chapter 12)

1. Express each of the following statements in algebraic form :
   (i) The sum of 3x and 4y is 8.
   (ii) 5x decreased by 7 gives y.
   (iii) 37 added to 4x gives 6x.
   (iv) 3x subtracted from 89 gives 44.

2. Group the **like terms** :
   (i) 7y, 3x, –8y, –x and $\frac{x}{5}$
   (ii) $3x^2, -5x^3, -x^2, 5x^2$ and $8x^3$
   (iii) $x^2y^3, -5x^3y^2, 8x^3y^2, -4x^2y^3$ and $-x^2y^3$

3. Write the *number of terms* in each of the following polynomials :
   (i) $5 + 4x \div 2$
   (ii) $5 + 4x + 2y$
   (iii) $8x^2 - 4x + 7$
   (iv) $\frac{x}{5} + \frac{x^2}{7} - \frac{x^3}{8} - \frac{1}{4}$
   (v) $6x^2 \div x - 18 \div 9 + x^2$

4. For each expression given below, state whether it is a monomial, or a binomial or a trinomial :
   (i) $x + y$
   (ii) $5x - 4y$
   (iii) $7x^2 + 5x + 8$
   (iv) $6a + 3 \div b$
   (v) $9 \div a \times b$
   (vi) $8a \div b$

5. Write the **coefficient of x^2y** in :
   (i) $-7x^2yz$
   (ii) $8abx^2y$
   (iii) $-x^2y$

6. Write the **coefficient of** :
   (i) x^2 in $-8x^2y$
   (ii) y in $-4y$
   (iii) x in $-xy^2$

7. Write the **numeral coefficient** in :
   (i) $7x^2y$
   (ii) $\frac{2x}{3}$
   (iii) $-\frac{5}{4}xy^2z$

8. Write the **degree** of each of the following polynomials :
   (i) $x^5 - 6x^8 + x$
   (ii) $4x^3 - x^4$
   (iii) $4 - x^2$
   (iv) $x - 1$
   (v) $x^2 + x - x^3$
   (vi) $x^3 - 8xy^2 + x^3y^3$
   (vii) $x^7 - 6y^4$
   (viii) $3y^3 - 2y^2z^4$
   (ix) $100x^8 - 8x^{100}$

9. Write each statement given below in algebraic form :
   (i) 28 more than twice of x is equal to 45.
   (ii) 3y reduced by 5z is greater than 8x.
   (iii) 6x divided by 13y is less than 17.
   (iv) 9 multiplied by 5x is equal to 2y.

10. State whether **true** or **false** :
    (i) If 23 is a constant and x is a variable, 23 + x is a constant.
    (ii) If 23 is a constant and x is a variable, 23x is a variable.
    (iii) If y is a variable and 57 is a constant, y – 57 is a variable.
    (iv) If 3x and 2y are variables, each of 3x + 2y, 3x – 2y, 3x ÷ 2y and 3x × 2y is a variable.