POWERS AND ROOTS

7.1 EXPONENTIAL NOTATIONS (Positive powers only)

If number 3 is multiplied to itself 4 times, we get : $3 \times 3 \times 3 \times 3$.

The product of $3 \times 3 \times 3 \times 3$ can be written as 3^4.

3^4 is called the exponential notation for the product of $3 \times 3 \times 3 \times 3$.

In the product $3 \times 3 \times 3 \times 3 = \mathbf{3^4}$, the repeating number **3** is called the **base**, whereas **4** is called the **exponent** (or *index* or *power*).

Similarly : (i) in 2^3, base = 2 and exponent (power) = 3

(ii) in $(-3)^4$, base = -3 and exponent (power) = 4

(iii) in $x^{\frac{2}{3}}$, base = x and exponent (index) = $\frac{2}{3}$ and so on.

In general, 3^4 is read as: 3 **raised to the power** 4 or **the fourth power of** three.

1. $(-3)^4 = -3 \times -3 \times -3 \times -3 = 81$ and $-(3)^4 = -(3 \times 3 \times 3 \times 3) = -81$
2. For any non-zero number a, $\mathbf{a^0 = 1}$

   i.e. $2^0 = 1$, $(-5)^0 = 1$, $\left(\frac{2}{3}\right)^0 = 1$ and so on.
3. For any number a, $\mathbf{a^1 = a}$

   i.e. $(-2)^1 = -2$, $5^1 = 5$, $\left(\frac{2}{3}\right)^1 = \frac{2}{3}$ and so on.

Example 1 :

Evaluate : (i) $3^2 \times 2^4$ (ii) $3^2 \times 2^0$

Solution :

(i) $3^2 \times 2^4 = 3 \times 3 \times 2 \times 2 \times 2 \times 2 = 144$ **(Ans.)**

(ii) $3^2 \times 2^0 = 3 \times 3 \times 2^0 = 9 \times 1 = 9$ **(Ans.)**

Example 2 :

Simplify and express the result in exponential form :

(i) $\frac{5 \times 2 \times 3^3}{3 \times 3 \times 2 \times 5^3}$ (ii) $\frac{3^4 \times 2^4}{2^3 \times 3^6}$

Solution :

(i) $\frac{5 \times 2 \times 3^3}{3 \times 3 \times 2 \times 5^3} = \frac{5 \times 2 \times 3 \times 3 \times 3}{3 \times 3 \times 2 \times 5 \times 5 \times 5} = \frac{3}{5 \times 5} = \frac{3}{5^2}$ **(Ans.)**

(ii) $\frac{3^4 \times 2^4}{2^3 \times 3^6} = \frac{3\times3\times3\times3\times2\times2\times2\times2}{2\times2\times2\times3\times3\times3\times3\times3\times3} = \frac{2}{3\times3} = \frac{2}{3^2}$ **(Ans.)**

Example 3 :

Evaluate : $6a^2b^3$ for $a = 3$ and $b = 2$.

Solution :

$6a^2b^3 = 6 \times a \times a \times b \times b \times b$

$= 6 \times 3 \times 3 \times 2 \times 2 \times 2 = 432$ **(Ans.)**

The following table will make it more clear :

Number	As a product of prime factors	In the exponential form
8	$2 \times 2 \times 2$	2^3
243	$3 \times 3 \times 3 \times 3 \times 3$	3^5
72	$2 \times 2 \times 2 \times 3 \times 3$	$2^3 \times 3^2$
630	$2 \times 3 \times 3 \times 5 \times 7$	$2^1 \times 3^2 \times 5^1 \times 7^1$
9000	$2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5$	$2^3 \times 3^2 \times 5^3$

EXERCISE 7(A)

1. Fill in the blanks :
   (i) In expression x^y, base = ... and exponent = ...
   (ii) In expression $(-6)^4$, power = ... and base = ...
   (iii) If base = 8 and exponent = 5, then expression = ...
   (iv) If base = –2 and power = 10, then expression = ...

2. Evaluate the following :
   (i) 5^0 = ...
   (ii) -5^0 = ...
   (iii) -4^3 = ...
   (iv) $(-1)^7$ = ...
   (v) If x = 1, then $15x^7$ = ...
   (vi) If y = 3, then y^3 = ...
   (vii) If a = –2, then $4a^3$ = ...
   (viii) If $3^a = 1$, then a = ...

3. Find the value of :
   (i) $(-2)^5$ (ii) $(-10)^3$ (iii) 10^4 (iv) 7^4

4. Find the value of :
   (i) $\left(\frac{2}{3}\right)^4$ (ii) $\left(-\frac{1}{2}\right)^6$ (iii) $\left(-\frac{2}{7}\right)^3$ (iv) $\left(\frac{3}{5}\right)^4$

5. Simplify and express the result in exponential notation :
   (i) $\frac{2\times2\times2\times2\times3}{3\times3\times3\times2}$ (ii) $\frac{3^5\times2^2}{3\times3\times2}$ (iii) $\frac{5\times5\times5^5\times3^2}{3\times3\times5^2}$
   (iv) $\frac{2^3\times3^4\times5^2}{5^4\times3^2\times2}$ (v) $\frac{5^6\times11^3}{11^5\times5\times5}$

6. Evaluate :
   (i) $\frac{(-3)^3\times(-2)^7}{(-2)^5}$ (ii) $8^5 \div 8^2$ (iii) $(-5)^7 \div 5^4$

7. Evaluate :

(i) a^3 for $a = 3$ (ii) $4b^3$ for $b = 4$
(iii) $2x^3$ for $x = 5$ (iv) $(3x)^2$ for $x = 1$
(v) $(4a)^3$ for $a = -1$ (vi) $2a^3b^2$ for $a = 2$ and $b = 3$
(vii) $3x^3y^4$ for $x = 1$ and $y = -1$ (viii) ab^3 for $a = 2$ and $b = -3$
(ix) $a^3 + b^3$ for $a = 1$ and $b = 2$ (x) $a^3 + b^3 - 3ab$ for $a = 2$ and $b = 1$

8. Express each of the following numbers in the exponential form of its prime factors :

(i) 16 (ii) 81 (iii) 144 (iv) 2700
(v) 3000 (vi) 6075 (vii) 4500

7.2 PROPERTIES OF EXPONENTS

Property 1 (Product Law) : $a^m \times a^n = a^{m+n}$

For example :

(i) $a^3 \times a^2 = a^{3+2} = a^5$ (ii) $3^7 \times 3^3 = 3^{7+3} = 3^{10}$

(iii) $(-5)^4 \times (-5)^2 = (-5)^6$ (iv) $\left(\frac{3}{4}\right)^6 \times \left(\frac{3}{4}\right)^5 = \left(\frac{3}{4}\right)^{11}$ and so on.

Property 2 (Quotient Law) :

$\frac{a^m}{a^n} = a^{m-n}$, if $m > n$ and $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$, if $m < n$ and $a \neq 0$.

For example :

(i) $\frac{a^5}{a^3} = a^{5-3} = a^2$ (ii) $\frac{a^3}{a^5} = \frac{1}{a^{5-3}} = \frac{1}{a^2}$

(iii) $\frac{3^7}{3^4} = 3^{7-4} = 3^3$ (iv) $\frac{3^4}{3^7} = \frac{1}{3^{7-4}} = \frac{1}{3^3}$ and so on.

For both the properties discussed above, the base must be the same.

Property 3 (Power Law) :

$(a^m)^n = a^{m \times n}$

For example :

(i) $(a^5)^3 = a^{5 \times 3} = a^{15}$ (ii) $(3^{-2})^5 = 3^{-2 \times 5} = 3^{-10}$ and so on.

Also, (i) $(a^3 \times b^4)^5 = a^{3 \times 5} \times b^{4 \times 5} = a^{15} \times b^{20}$

(ii) $\left(\frac{3^2}{2^4}\right)^3 = \frac{3^{2\times 3}}{2^{4\times 3}} = \frac{3^6}{2^{12}}$ and so on.

$(a \times b)^5 = a^5 \times b^5$, but $(a + b)^5 \neq a^5 + b^5$ and $(a - b)^5 \neq a^5 - b^5$.

Example 4 :

Using the properties of exponents, evaluate :(i) $\frac{2^3 \times 2^7}{2^6}$ (ii) $\frac{(-2)^3 \times 2^7}{2^6}$ (iii) $\frac{3^8 \times 4^3}{3^6 \times 4^4}$

Solution : (i) $\frac{2^3 \times 2^7}{2^6} = \frac{2^{3+7}}{2^6} = \frac{2^{10}}{2^6}$

$= 2^{10-6} = 2^4 = 2 \times 2 \times 2 \times 2 = 16$ **(Ans.)**

(ii) $\frac{(-2)^3 \times 2^7}{2^6} = \frac{-2^3 \times 2^7}{2^6}$ $[(-2)^3 = -2 \times -2 \times -2 = -2 \times 2 \times 2 = -2^3]$

$= \frac{-2^{10}}{2^6} = -2^{10-6} = -2^4 = -2 \times 2 \times 2 \times 2 = -16$ **(Ans.)**

(iii) $\frac{3^8 \times 4^3}{3^6 \times 4^4} = \frac{3^{8-6}}{4^{4-3}} = \frac{3^2}{4^1} = \frac{3 \times 3}{4} = \frac{9}{4} = 2\frac{1}{4}$ **(Ans.)**

EXERCISE 7(B)

1. Fill in the blanks :

(i) $x^a \times x^b =$ .... (ii) $a^3 \times a^8 =$ .... (iii) $a^5 \times b^3 =$ ....

(iv) $2^3 \times 3^2 =$ .... (v) $\frac{5^7}{5^4} =$ .... (vi) $\frac{5^3}{5^5} =$ ....

(vii) $\frac{8^6}{8^4} =$ .... (viii) $(a^3 \times b^4)^2 =$ .... (ix) $\left(\frac{3^0}{2^2}\right)^2 =$ ....

(x) $(4^2 \times 5^0)^3 =$ ....

2. Using the properties of exponents, evaluate :

(i) $\frac{3^4 \times 3^9}{3^{15}}$ (ii) $\frac{4^6 \times 4^3}{4^5 \times 4^2}$ (iii) $\frac{(-2)^6 \times (-2)^5}{(-2)^7}$

(iv) $\frac{2^3 \times 5^2}{5^4}$ (v) $\frac{5^2 \times 3^5}{3^3}$

3. Evaluate :

(i) $\frac{(-3)^3 \times (-2)^4}{(-2)^2}$ (ii) $\frac{(2^3 \times 5^4)^2}{2^4 \times 5^5}$ (iii) $(-5)^4 \div 5^4$ (iv) $(-3)^7 \div (-3)^6$

4. Evaluate :

(i) $2^3 - 3^2 + 4^0$ (ii) $5^2 + 2^2 - 3^3 + 8^0$

5. Evaluate :

(i) $2x^2$, if $x = 3$ (ii) x^3y^2, if $x = 2$ and $y = 3$

(iii) $x^2 + y^3$, if $x = 1$ and $y = 2$

(iv) $3x^2y - 2xy^2$, if $x = 5$ and $y = 6$

(iv) $\mathbf{3x^2y - 2xy^2} = 3 \times 5^2 \times 6 - 2 \times 5 \times 6^2$

$= 3 \times 25 \times 6 - 10 \times 36$

$= 450 - 360 = \mathbf{90}$

7.3 SQUARES

When a number is multiplied by itself, the product obtained is called the *square* of that number.

For example :

(i) Since $4 \times 4 = 16$, $\therefore$ 16 is square of 4, and we write : $(4)^2 = 16$

(ii) Since $-2 \times -2 = 4$, $\therefore$ 4 is square of -2 and we write : $(-2)^2 = 4$

(iii) Since $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, $\therefore$ $\frac{4}{9}$ is square of $\frac{2}{3}$ which is written as $\left(\frac{2}{3}\right)^2 = \frac{4}{9}$

(iv) Since $0.2 \times 0.2 = 0.04$ $\therefore$ 0.04 is square of 0.2 *i.e.* $(0.2)^2 = 0.04$ *and so on.*

Whether the number is positive or negative, its square is always positive.

e.g. (i) $(3)^2 = 3 \times 3 = 9$

(ii) $(-3)^2 = -3 \times -3 = 9$

(iii) $(-5)^2 = -5 \times -5 = 25$ and so on

More examples :

(i) Square of $0 = 0^2 = \mathbf{0}$

(ii) Square of $5 = 5^2 = 5 \times 5 = \mathbf{25}$

(iii) Square of $-\frac{2}{5} = \left(-\frac{2}{5}\right)^2 = \left(-\frac{2}{5}\right) \times \left(-\frac{2}{5}\right) = \mathbf{\frac{4}{25}}$

(iv) Square of $2\frac{3}{7} = \left(2\frac{3}{7}\right)^2 = \left(\frac{17}{7}\right)^2 = \frac{17}{7} \times \frac{17}{7} = \frac{289}{49} = \mathbf{5\frac{44}{49}}$

(v) Square of $-3\frac{1}{2} = \left(-3\frac{1}{2}\right)^2 = \left(-\frac{7}{2}\right)^2 = \left(-\frac{7}{2}\right) \times \left(-\frac{7}{2}\right) = \frac{49}{4} = \mathbf{12\frac{1}{4}}$

(vi) Square of $-2.3 = (-2.3)^2 = -2.3 \times -2.3 = \mathbf{5.29}$ and so on.

EXERCISE 7(C)

1. Find the squares of first five natural numbers.
2. Find the squares of first six even natural numbers.
3. Find the squares of first four odd natural numbers.
4. Find the squares of first five prime numbers.
5. Find the squares of :

   (i) 9 (ii) $\frac{2}{5}$ (iii) $1\frac{2}{7}$ (iv) $2\frac{3}{4}$

6. Find the squares of :

   (i) -3 (ii) $-\frac{2}{3}$ (iii) $-1\frac{2}{5}$ (iv) $-2\frac{1}{4}$

7. Find the squares of :

   (i) 2.5 (ii) 0.6 (iii) 0.23
   (iv) 0.02 (v) -1.6 (vi) -0.8

7.4 SQUARE ROOT

The *square root* of a given number, is that number which when multiplied by itself, gives the given number.

For example :

(i) Square root of 16 = 4, as 4 multiplied by itself = 4 × 4 = 16.

(ii) Square root of 25 = 5, as 5 multiplied by itself = 5 × 5 = 25 and so on.

7.5 SYMBOL FOR SQUARE ROOT

The square root is denoted by the radical sign $\sqrt{\ }$.

For example :

(i) $\sqrt{9}$ means square root of 9; (ii) Square root of $\frac{16}{25} = \sqrt{\frac{16}{25}}$

(iii) $\sqrt{0 \cdot 16}$ means square root of 0·16 and so on.

7.6 METHODS OF FINDING THE SQUARE ROOT

(a) ***To find the square root of a number*** *(whole number)* ***using the prime factor method :***

Steps :

1. Express the given number as the product of its prime factors.
2. Make groups, each consisting of two identical factors obtained in step (1).
3. Take one factor from each group and multiply them together.
4. The product so obtained is the square root of the given number.

Example 5 :

Find the square root of : (i) 144 (ii) 225 (iii) 900

Solution :

(i) Since 144 = 2 × 2 × 2 × 2 × 3 × 3

$\therefore$ **Square root of 144** $= \sqrt{144}$

$= \sqrt{(2\times2)\times(2\times2)\times(3\times3)}$

$= 2 \times 2 \times 3$ [Taking one factor from each pair]

$= 12$ **(Ans.)**

OR, directly, as $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$= (2 \times 2) \times (2 \times 2) \times (3 \times 3)$

$\therefore$ **Square root of 144** $= 2 \times 2 \times 3 = 12$ **(Ans.)**

(ii) $\because$ $225 = 3 \times 3 \times 5 \times 5$

$= (3 \times 3) \times (5 \times 5)$

$\therefore$ **Square root of 225** $= 3 \times 5 = 15$ **(Ans.)**

(iii) $= \sqrt{2\times2\times3\times3\times5\times5}$

$\sqrt{900} = \sqrt{2\times2\times3\times3\times5\times5} = 2 \times 3 \times 5 = 30$ **(Ans.)**

(b) ***To find the square root of a number in the fraction form :***

The square root of a fraction is found by getting the square roots of its numerator and denominator separately.

Square root of a fraction = $\dfrac{\text{Square root of its numerator}}{\text{Square root of its denominator}}$ *i.e.* $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

For example :

(i) $\sqrt{\dfrac{225}{49}} = \sqrt{\dfrac{3\times3\times5\times5}{7\times7}} = \dfrac{3\times5}{7} = \dfrac{15}{7} = \mathbf{2\frac{1}{7}}$

(ii) $\sqrt{4\cdot41} = \sqrt{\dfrac{441}{100}} = \sqrt{\dfrac{3\times3\times7\times7}{2\times2\times5\times5}} = \dfrac{3\times7}{2\times5} = \dfrac{21}{10} = \mathbf{2\cdot1}$

(c) ***To find the square root of a number in index form :***

In general, the square root of a given number in its index form can be obtained only when its index (power) is an even number *i.e.* the given number is of the form 2^{10}, 3^8, 5^4, etc.

Method : Keeping the base same, divide index (power) by 2.

Example 6 :

Find the square root of : (i) 2^8 (ii) 3^6 (iii) $2^4 \times 5^2$

Solution :

(i) 2^8 shows that the power of 2 is 8, and half of 8 is 4.

∴ **Square root of 2^8** $= 2^4 = 2 \times 2 \times 2 \times 2 = \mathbf{16}$ **(Ans.)**

OR, directly, $\sqrt{2^8} = 2^4 = 2 \times 2 \times 2 \times 2 = \mathbf{16}$ **(Ans.)**

Similarly :

(ii) $\sqrt{3^6} = 3^3 = 3 \times 3 \times 3 = \mathbf{27}$ **(Ans.)**

(iii) $\sqrt{2^4 \times 5^2} = 2^2 \times 5^1 = 2 \times 2 \times 5 = \mathbf{20}$ **(Ans.)**

More examples :

(i) $\sqrt{44\times22\times8} = \sqrt{(2\times2\times11)\times(2\times11)\times(2\times2\times2)}$

$= \sqrt{(2\times2)\times(11\times11)\times(2\times2)\times(2\times2)}$

$= 2 \times 11 \times 2 \times 2 = \mathbf{88}$

(ii) $\sqrt{\dfrac{3^2\times36}{5^4}} = \sqrt{\dfrac{3^2\times(2\times2)\times(3\times3)}{5^4}} = \dfrac{3^1\times2\times3}{5^2} = \mathbf{\dfrac{18}{25}}$ and so on.

EXERCISE 7(D)

1. Fill in the blanks :

(i) The square of 7 is $(7)^a$, then a = ...

(ii) The square of b is 49, then b = ...

(iii) If x is positive and $x^2 = 121$, then x = ...

(iv) If the square of 0·8 = 0·64, then the square root of 0·64 = ...

2. Find the square root of :

(i) 64 (ii) 144 (iii) 225 (iv) 324

(v) 441 (vi) 484 (vii) 625 (viii) 729

3. Find the square root of :

(i) $\frac{4}{9}$ (ii) $\frac{9}{16}$ (iii) $\frac{25}{64}$

(iv) $1\frac{7}{9}$ (v) $1\frac{9}{16}$ (vi) $2\frac{2}{49}$

4. Find the square root of :

(i) 4×49 (ii) 9×81 (iii) 100×16

(iv) $1 \times 25 \times 36$ (v) $36 \times 64 \times 81$ (vi) $\frac{81 \times 121}{225}$

5. Find the square root of :

(i) 98×8 (ii) 72×18 (iii) $8 \times 25 \times 200$

6. Evaluate :

(i) $\sqrt{144 \times 4}$ (ii) $\sqrt{25 \times 400}$ (iii) $\sqrt{100 \times 36}$

(iv) $\sqrt{9 \times 81 \times 100}$ (v) $\sqrt{16 \times 25 \times 4 \times 64}$ (vi) $\sqrt{81 \times 64 \times 4}$

(vii) $\sqrt{2^2 \times 4^2 \times 3^2}$ (viii) $\sqrt{3^6 \times 5^4 \times 2^8}$ (ix) $\sqrt{\frac{196 \times 100}{400}}$

(x) $\sqrt{\frac{2^8 \times 3^2}{5^2}}$ (xi) $\sqrt{\frac{1^2 \times 25}{4^2}}$ (xii) $\sqrt{\frac{81 \times 225}{100}}$

7. Evaluate :

(i) $\sqrt{25} + \sqrt{16}$ (ii) $\sqrt{49} - \sqrt{36}$ (iii) $\sqrt{25} \times \sqrt{16}$

(iv) $\sqrt{144} \div \sqrt{16}$ (v) $\sqrt{49} - \sqrt{16} + \sqrt{4}$ (vi) $\frac{\sqrt{81} + \sqrt{9}}{\sqrt{196} - \sqrt{64}}$

7.7 CUBES

When a **number** is **multiplied** by itself **three times**, the product obtained is called the **cube** of that number.

For example :

(i) Since $4 \times 4 \times 4 = 64$, $\therefore$ 64 is the cube of 4, and we write : $(4)^3 = 64$

(ii) Since $-2 \times -2 \times -2 = -8$, $\therefore$ -8 is the cube of -2, and we write : $(-2)^3 = -8$

(iii) Since $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$, $\therefore$ $\frac{8}{27}$ is the cube of $\frac{2}{3}$, and we write : $\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

(iv) Since $0.2 \times 0.2 \times 0.2 = 0.008$,

$\therefore$ 0.008 is cube of 0.2, and we write : $(0.2)^3 = 0.008$

1. The cube of a positive number is always positive.
   e.g., $(3)^3 = 3 \times 3 \times 3 = 27$, $(5)^3 = 5 \times 5 \times 5 = 125$ and so on.
2. The cube of a negative number is always negative.
   e.g., $(-3)^3 = -3 \times -3 \times -3 = -27$, $(-5)^3 = -5 \times -5 \times -5 = -125$ and so on.
3. $-a^3 = (-a)^3$
   i.e., $-2^3 = (-2)^3$, $-5^3 = (-5)^3$ and so on.

More examples :

(i) The cube of $0 = 0^3 = 0 \times 0 \times 0 = \mathbf{0}$

(ii) The cube of $6 = 6^3 = 6 \times 6 \times 6 = \mathbf{216}$

(iii) The cube of $-\frac{2}{3} = \left(-\frac{2}{3}\right)^3 = -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} = \mathbf{-\frac{8}{27}}$

(iv) The cube of $2\frac{2}{3} = \left(\frac{8}{3}\right)^3 = \frac{8}{3} \times \frac{8}{3} \times \frac{8}{3} = \frac{512}{27} = \mathbf{18\frac{26}{27}}$

(v) The cube of $-3\frac{1}{2} = \left(-3\frac{1}{2}\right)^3 = \left(-\frac{7}{2}\right)^3 = -\frac{7}{2} \times -\frac{7}{2} \times -\frac{7}{2} = -\frac{343}{8} = \mathbf{-42\frac{7}{8}}$

(vi) The cube of $1.2 = (1.2)^3 = 1.2 \times 1.2 \times 1.2 = 1.728$ and so on.

EXERCISE 7(E)

1. Find the cubes of the first three whole numbers.
2. Find the cubes of each natural number between 3 and 8
3. Find the cubes of each integer between – 3 and 3.
4. Find the cubes of each integer from – 5 to – 2.
5. Find the cubes of the even natural numbers between 4 and 10.
6. Find the cubes of the odd natural numbers from 3 to 9.
7. Find the cubes of :
   (i) $\frac{4}{5}$ (ii) $2\frac{3}{4}$ (iii) $2\frac{1}{2}$ (iv) $3\frac{1}{3}$
8. Find the cubes of :
   (i) $-\frac{2}{5}$ (ii) $-1\frac{2}{5}$ (iii) $-2\frac{1}{4}$ (iv) $-3\frac{1}{3}$
9. Find the cubes of :
   (i) 0.1 (ii) 0.5 (iii) 1.5 (iv) 0.02
   (v) – 0.6 (vi) – 0.08 (vii) – 0.14 (viii) – 1.6

7.8 CUBE ROOT

The *cube root* of a given number, is that number which when multipled by itself three times, gives the given number.

For example :

(i) The cube root of 64 = 4, as $4 \times 4 \times 4 = 64$

(ii) The cube root of 125 = 5, as $5 \times 5 \times 5 = 125$

(iii) The cube root of $\frac{8}{27} = \frac{2}{3}$, as $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$

(iv) The cube root of $-\frac{1}{8} = -\frac{1}{2}$, as $-\frac{1}{2} \times -\frac{1}{2} \times -\frac{1}{2} = -\frac{1}{8}$

(v) The cube root of $0.027 = 0.3$, as $0.3 \times 0.3 \times 0.3 = 0.027$ and so on.

7.9 SYMBOL FOR THE CUBE ROOT

The symbol for cube root is $\sqrt[3]{\ }$.

That is, the cube root of 243 is written as : $\sqrt[3]{243}$ or $(243)^{\frac{1}{3}}$

Thus :

(i) The cube root of 64 = 4 $\Rightarrow$ $\sqrt[3]{64} = 4$

(ii) The cube root of $\frac{8}{27} = \frac{2}{3}$ $\Rightarrow$ $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$

(iii) The cube root of $-\frac{1}{8} = -\frac{1}{2}$ $\Rightarrow$ $\sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$

(iv) The cube root of 0.027 = 0.3 $\Rightarrow$ $\sqrt[3]{0.027} = 0.3$ and so on.

7.10 METHODS OF FINDING THE CUBE ROOT

(a) *To find the cube root of a number* (integer) ***using the factor method :***

Steps :

1. Express the given number as the product of its prime factors.
2. Make groups [out of the prime factors obtained in step (1)], each consisting of three identical factors.
3. Take one factor from each group and multiply them together.
4. The product so obtained is the cube root of the given number.

Example 7 :

Find the cube root of : (i) 64 (ii) 216 (iii) 729

Solution :

> The cube root of a positive number is always positive.

(i) Since, $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$\therefore$ **The cube root of 64** $= \sqrt[3]{64}$

$= \sqrt[3]{2\times2\times2\times2\times2\times2}$

$= \sqrt[3]{(2\times2\times2)\times(2\times2\times2)}$

$= 2 \times 2 = 4$ **(Ans.)**

OR, Since, $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$= (2 \times 2 \times 2) \times (2 \times 2 \times 2)$

$\therefore$ **The cube root of 64** $= 2 \times 2 = 4$ **(Ans.)**

(ii) Since, $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$
$= (2 \times 2 \times 2) \times (3 \times 3 \times 3)$

∴ **The cube root of 216** $= 2 \times 3 = 6$ **(Ans.)**

(iii) Since, $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= (3 \times 3 \times 3) \times (3 \times 3 \times 3)$

∴ **The cube root of 729** $= 3 \times 3 = 9$ **(Ans.)**

Example 8 :

Find the cube root of : (i) – 64 (ii) – 3375

Solution :

> The cube root of a negative number is always negative.

(i) Since, $-64 = -2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= -(2 \times 2 \times 2) \times (2 \times 2 \times 2)$

∴ **The cube root of – 64** $= -2 \times 2 = -4$ **(Ans.)**

(ii) Since, $-3375 = -5 \times 5 \times 5 \times 3 \times 3 \times 3$
$= -(5 \times 5 \times 5) \times (3 \times 3 \times 3)$

∴ **The cube root of – 3375** $= -5 \times 3 = -15$ **(Ans.)**

(b) To find the cube root of a number in the fraction form :

> The cube root of a fraction = $\dfrac{\text{The cube root of its numerator}}{\text{The cube root of its denominator}}$

For example :

(i) The cube root of $\dfrac{8}{27} = \dfrac{\text{Cube root of 8}}{\text{Cube root of 27}}$

$= \dfrac{2}{3}$ [$8 = 2 \times 2 \times 2$ and $27 = 3 \times 3 \times 3$]

i.e. $\sqrt[3]{\dfrac{8}{27}} = \dfrac{\sqrt[3]{8}}{\sqrt[3]{27}} = \dfrac{\sqrt[3]{2 \times 2 \times 2}}{\sqrt[3]{3 \times 3 \times 3}} = \dfrac{2}{3}$

(ii) Since $0.729 = \dfrac{729}{1000} = \dfrac{3 \times 3 \times 3 \times 3 \times 3 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5} = \dfrac{(3 \times 3 \times 3) \times (3 \times 3 \times 3)}{(2 \times 2 \times 2) \times (5 \times 5 \times 5)}$

∴ The cube root of $0.729 = \dfrac{3 \times 3}{2 \times 5} = \dfrac{9}{10} = 0.9$

(iii) Since $-0{\cdot}027 = -\dfrac{27}{1000} = -\dfrac{3 \times 3 \times 3}{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

The cube root of – 0·027 $= -\dfrac{3}{2 \times 5} = -\dfrac{3}{10} = -0{\cdot}3$

(c) To find the cube root of a number in index form :

In general, the cube root of a given number, in the index form, is obtained only when its index (power) is exactly divisible by 3.

Method : Keeping the base same, divide the index (power) by 3.

Example 9 :

Find the cube root of : (i) 2^6 (ii) 3^9 (iii) 5^3

Solution :

(i) The cube root of $2^6 = 2^{6 \div 3}$
$= 2^2 = 2 \times 2 = 4$ **(Ans.)**

i.e. $\sqrt[3]{2^6} = 2^{6 \div 3} = 2^2 = 2 \times 2 = 4$ **(Ans.)**

(ii) The cube root of $3^9 = 3^{9 \div 3}$
$= 3^3 = 3 \times 3 \times 3 = 27$ **(Ans.)**

(iii) The cube root of $5^3 = 5^{3 \div 3}$
$= 5^1 = 5$ **(Ans.)**

Example 10 :

Find the cube root of : (i) $2^6 \times 3^3$ (ii) $5^6 \times 2^9$

Solution :

Method : Divide the power (index) of each number used by 3.

(i) The cube root of $2^6 \times 3^3 = 2^{6 \div 3} \times 3^{3 \div 3}$
$= 2^2 \times 3^1 = 2 \times 2 \times 3 = 12$ **(Ans.)**

(ii) The cube root of $5^6 \times 2^9 = 5^{6 \div 3} \times 2^{9 \div 3}$
$= 5^2 \times 2^3 = 5 \times 5 \times 2 \times 2 \times 2 = 200$ **(Ans.)**

OR, directly,

Dividing each power by 3

(i) $\sqrt[3]{2^6 \times 3^3} = 2^2 \times 3$
$= 2 \times 2 \times 3 = 12$ **(Ans.)**

(ii) $\sqrt[3]{5^6 \times 2^9} = 5^2 \times 2^3$
$= 5 \times 5 \times 2 \times 2 \times 2 = 200$ **(Ans.)**

The cube root of a^3b^3 = ab and the cube root of $\frac{a^3}{b^3} = \frac{a}{b}$, but the cube root of $a^3 + b^3 \neq a + b$ and the cube root of $a^3 - b^3 \neq a - b$.

More examples :

(i) $\sqrt[3]{24 \times 45 \times 25} = \sqrt[3]{(2 \times 2 \times 2 \times 3) \times (3 \times 3 \times 5) \times (5 \times 5)}$
$= \sqrt[3]{(2 \times 2 \times 2) \times (3 \times 3 \times 3) \times (5 \times 5 \times 5)}$
$= 2 \times 3 \times 5 = 30$

(ii) $\sqrt[3]{\frac{50 \times 36 \times 75}{1715}} = \sqrt[3]{\frac{(5 \times 5 \times 2) \times (2 \times 2 \times 3 \times 3) \times (3 \times 5 \times 5)}{5 \times 7 \times 7 \times 7}}$
$= \sqrt[3]{\frac{(5 \times 5 \times 5) \times (2 \times 2 \times 2) \times (3 \times 3 \times 3) \times 5}{5 \times (7 \times 7 \times 7)}}$
$= \frac{5 \times 2 \times 3}{7} = \frac{30}{7} = 4\frac{2}{7}$

EXERCISE 7(F)

1. Fill in the blanks :
   (i) The cube of 5 is 5^x, then $x =$ .... (ii) The cube of y is 64, then $y =$ ....
   (iii) If $x^3 = 125$, then $x =$ .... (iv) If $a^a = 27$, then $a =$ ....
   (v) If cube of 0.5 is 0.125, then cube root of 0.125 = ...
2. Find the cube root of :
   (i) 1 (ii) 343 (iii) 512 (iv) 3375
3. Find the cube root of :
   (i) –1 (ii) –125 (iii) –216 (iv) –512
4. Find the cube root of :
   (i) $\frac{1}{8}$ (ii) $3\frac{3}{8}$ (iii) $2\frac{10}{27}$ (iv) $-\frac{8}{27}$
5. Find the cube root of :
   (i) 0.125 (ii) 0.064 (iii) 0.001 (iv) 3.375
   (v) – 0.008 (vi) – 0.064
6. Find the cube root of :
   (i) 5^6 (ii) 3^{15} (iii) $2^9 \times 3^{12}$ (iv) $4^6 \times 3^9 \times 2^{12}$

Revision Exercise (Chapter 7)

1. Fill in the blanks :
   (i) $5^0 =$ .... (ii) $(-5)^0 =$ .... (iii) $-5^0 =$ .... (iv) $5^1 =$ ....
   (v) $(-5)^1 =$ .... (vi) $-5^1 =$ .... (vii) $5^2 =$ .... (viii) $(-5)^2 =$ ....
   (ix) $-5^2 =$ .... (x) $5^3 =$ .... (xi) $(-5)^3 =$ .... (xii) $-5^3 =$ ....
2. State **true** or **false** :
   (i) 2^8 = a positive number (ii) -2^8 = a positive number
   (iii) $(-2)^8$ = a negative number (iv) 2^5 = a negative number
   (v) -2^5 = a negative number (vi) $(-2)^5$ = a positive number
3. Verify that :
   (i) $5^2 - 3^2$ and 4^2 are equal. (ii) $10^2 - 8^2$ and 6^2 are equal.
   (iii) $12^2 + 5^2$ and 13^2 are equal.
4. Hari planted 324 plants in such a way that there were as many rows of plants as there were number of columns. Find the number of rows and columns.
5. Evaluate :
   (i) $\sqrt{64} - \sqrt[3]{64}$ (ii) $\sqrt[3]{125} + \sqrt{81}$ (iii) $\sqrt[3]{27} - \sqrt{49} + \sqrt[3]{216}$
6. (i) Write the number whose cube root is 0·6.
   (ii) Write the number whose cube root is –0·6.
7. Evaluate :
   (i) $8^0 \times 3^2 + 2^3 \times 5$ (ii) $5^2 - 9^2 + 3^3 \times 12^0 \times 2^2$
   (iii) $8^2 \times 6^0 + 2^3 \times 3^2$ (iv) $6^2 \times 10^0 - (-5)^0 + 4$
8. Evaluate :
   (i) $\sqrt{3 + \sqrt{169}}$ (ii) $\sqrt{9 - \sqrt{25}}$ (iii) $\sqrt{7 + \sqrt{81}}$