CHAPTER 4

FACTORS AND MULTIPLES

(Including H.C.F. and L.C.M.)

4.1 FACTORS

When two or more natural numbers are multiplied together, the result is referred to as their **product**, and each of the numbers multiplied is called a **factor** of this product.

For example :

(i) Product of 5 and 7 = 5 × 7 = 35; therefore 5 and 7 are factors of 35.

(ii) Product of 2, 3 and 7 = 2 × 3 × 7 = 42; therefore each of 2, 3 and 7 is a factor of 42.

In other words :

Any natural number that divides a given natural number completely is called a factor of the given number.

For example :

(i) 5 divides 20 completely ⇒ 5 is a factor of 20

(ii) 6 divides 12 completely ⇒ 6 is a factor of 12

(iii) 15 divides 30 completely ⇒ 15 is a factor of 30 and so on.

Now consider the following examples :

(i) 24 = 1 × 24 ⇒ 1 and 24 are factors of 24.

(ii) 24 = 2 × 12 ⇒ 2 and 12 are factors of 24.

(iii) 24 = 3 × 8 ⇒ 3 and 8 are factors of 24.

(iv) 24 = 4 × 6 ⇒ 4 and 6 are factors of 24.

Combining, we get :

1, 2, 3, 4, 6, 8, 12 and 24 are factors of 24.

∴ Factors of 24 = F_{24}

= 1, 2, 3, 4, 6, 8, 12 and 24

Each of 1, 2, 3, 4, 6, 8, 12 and 24 divides 24 completely.

In the same way,

(i) F_{30} = Factors of 30

= 1, 2, 3, 5, 6, 10, 15 and 30

(ii) F_{18} = 1, 2, 3, 6, 9 and 18

(iii) F_{45} = 1, 3, 5, 9, 15 and 45 and so on.

Factors of 6
= Each natural number that divides 6 completely
= 1, 2, 3 and 6

- 1 (one) is a factor of every number.
- Every number is a factor of itself.

Example 1 :

Write the factors of (i) 13 (ii) 25 (iii) 28

Solution :

(i) Factors of 13 = F_{13} = 1 and 13 (Ans.)

(ii) F_{25} = 1, 5 and 25 (Ans.)

(iii) F_{28} = 1, 2, 4, 7, 14 and 28 (Ans.)

4.2 PRIME NUMBERS

A natural number that is divisible only by 1 (one) and itself is called a ***prime number****.*

For example :

(i) 2 is divisible only by 1 (one) and itself; therefore **2 is a prime number**.

(ii) 3 is divisible only by 1 and itself; therefore **3 is a prime number**.

(iii) 4 is divisible by 1 and itself as well as by 2; so **4 is not a prime number**.

In the same way; we find :

(iv) each of 5, 7, 11, 13, 17, ... is a prime number.

(v) none of 1, 4, 6, 8, 9, 10, 12, ... is a prime number.

1. *Every prime number is greater than 1 (one).*
2. *Two (2) is the smallest prime number.*
3. *All prime numbers except 2 are odd.*

To be more clear, note :

If a natural number has **only two factors**, it is a **prime number**.

⇒ (i) **1** (one) is **not a prime number** as it has only **one factor**, 1, which is itself.

(ii) 7 is a **prime number** as it has only **two factors**, 1 and 7.

(iii) **10** is **not a prime number** as it has **more than two** factors : 1, 2, 5 and 10.

Every natural number that has more than two factors is called a **composite number**.

Since, factors of 10 are 1, 2, 5 and 10

∴ **10 is a composite number**

Similarly, each of 4, 6, 8, 9, 10, 12, 14, ..., 24, 25, 26, 27, etc. is a composite number.

Every even number greater than 2 is a composite number.

4.3 PRIME NUMBERS FROM 1 TO 100

ERATOSTHENES, a Greek scholar, used the following method to distinguish the prime numbers from among the natural numbers. For this reason, this method is known as the **Sieve of Eratosthenes**.

Step 1 : Write the natural numbers 1 to 100 in rows of 10 as shown below :

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Step 2 : 1 (one) is not a prime number; strike it out.

Step 3 : The first, *i.e.* the smallest, prime number is 2; encircle 2 and strike out all other numbers which are divisible by 2.

Step 4 : The next prime number is 3; encircle 3 and strike out all other numbers that are divisible by 3.

(Some of these numbers are already cut as they are divisible by 2 as well).

Step 5 : The next prime number is 5; encircle 5 and strike out all other numbers that are divisible by 5.

(Some of these numbers are already cut as they are divisible by 2 and/or 3).

Step 6 : Adopt the same steps for prime numbers 7, 11, 13 and so on.

Finally, some numbers emerge encircled, and they are the prime numbers between 1 and 100.

4.4 PRIME FACTORS

Let us take a number, say, 24. The factors of 24 are : 1, 2, 3, 4, 6, 8, 12 and 24. Out of these factors, 2 and 3 are prime numbers, and so the factors **2 and 3** are called the **prime factors of 24**.

Thus, factors of 24 = 1, 2, 3, 4, 6, 8, 12 and 24

$\Rightarrow$ Prime factors of 24 = 2 and 3

In other words, we write :

F_{24} = 1, 2, 3, 4, 6, 8, 12 and 24; and $P.F_{24}$ = 2 and 3

In the same way :

(i) F_{50} = 1, 2, 5, 10, 25 and 50 $\Rightarrow$ $P.F_{50}$ = 2 and 5

(ii) F_{64} = 1, 2, 4, 8, 16, 32 and 64 $\Rightarrow$ $P.F_{64}$ = 2 and so on.

EXERCISE 4(A)

1. Write all the factors of :
   (i) 15 (ii) 55 (iii) 48 (iv) 36 (v) 84
2. Write all prime numbers :
   (i) less than 25 (ii) between 15 and 35 (iii) between 8 and 76
3. Write the prime numbers from :
   (i) 5 to 45 (ii) 2 to 32 (iii) 8 to 48 (iv) 9 to 59
4. Write the prime factors of :
   (i) 16 (ii) 27 (iii) 35 (iv) 49
5. If P_n means prime factors of n, find :
   (i) P_6 (ii) P_{24} (iii) P_{50} (iv) P_{42}

4.5 HIGHEST COMMON FACTOR

H.C.F. stands for **Highest Common Factor**, and the H.C.F. of two or more given numbers is the greatest number that completely divides each of the given numbers.

For example :

(i) The greatest number that can divide both 18 and 24 completely is 6; therefore, H.C.F. of 18 and 24 = 6.

(ii) H.C.F. of numbers 16, 24 and 32 is 8; this is because 8 is the greatest number that divides each of the given numbers 16, 24 and 32 completely.

4.6 METHODS OF FINDING H.C.F.

For finding the H.C.F. of two or more given numbers, any of the following three methods can by used :

1. *Common Factor Method* 2. *Prime Factor Method*
3. *Division Method*

4.7 COMMON FACTOR METHOD

1. Find all the possible factors of each given number.
2. From the factors obtained in Step 1, select the common factors.
3. Out of the common factors, obtained in Step 2, take the highest factor, which is the **Highest Common Factor** (H.C.F.) of the given numbers.

Example 2 :

Using the common factor method, find the H.C.F. of 36 and 48.

Solution :

Step 1 : Factors of 36, *i.e.* F_{36} = 1, 2, 3, 4, 6, 9, 12, 18 and 36
Similarly, F_{48} = 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48

Step 2 : Factors that are common to F_{36} and F_{48}
= 1, 2, 3, 4, 6 and 12

Step 3 : From the result of Step 2, the highest common factor = 12

⇒ **H.C.F.** of the given numbers 36 and 48 = **12** **(Ans.)**

12 is the largest number that divides both 36 and 48 completely.

Example 3 :

Using the common factor method, find the H.C.F. of :

(i) 18, 27 and 36 (ii) 16, 32 and 49

Solution :

(i) F_{18} = 1, 2, 3, 6, 9 and 18

F_{27} = 1, 3, 9 and 27

and F_{36} = 1, 2, 3, 4, 6, 9, 12, 18 and 36

∴ Common factors = 1, 3 and 9

⇒ Required **H.C.F. = 9** **(Ans.)**

(ii) F_{16} = 1, 2, 4, 8 and 16

F_{32} = 1, 2, 4, 8, 16 and 32

and F_{49} = 1, 7 and 49

∴ Common factor = 1

⇒ Required **H.C.F. = 1** **(Ans.)**

1 is the greatest number that divides each of 16, 32 and 49 completely.

4.8 PRIME FACTOR METHOD

Steps :

1. Split each given number into its prime factors.
2. Select the common prime factors.
3. Multiply the prime factors obtained in Step 2.

The product so obtained is the H.C.F. of the given numbers.

Example 4 :

Find the H.C.F. of 15 and 25.

5 and 3 are primes

Solution :

Prime factors of 15 are 5 and 3, since 15 = 5 × 3

Prime factors of 25 are 5 and 5, since 25 = 5 × 5

Since the common prime factor is 5 only,

∴ **H.C.F.** of 15 and 25 = **5** **(Ans.)**

Example 5 :

Find the H.C.F. of 24, 12, 36 and 60.

Solution :

Since 24 = 2 × 2 × 2 × 3; 12 = 2 × 2 × 3;

36 = 2 × 2 × 3 × 3 and 60 = 2 × 2 × 3 × 5.

The prime factors common to the given numbers are 2, 2 and 3.

∴ **H.C.F.** = 2 × 2 × 3 = **12** **(Ans.)**

1. Any two numbers that do not have a common prime factor are called **co-prime numbers**.
   e.g. (i) 39 *and* 175 (ii) 15 *and* 16 (iii) 27 *and* 64 and so on

Reason :

$39 = 3 \times 13$ *and* $175 = 5 \times 5 \times 7$

$\Rightarrow$ 39 *and* 175 *have no common factor.*

$\therefore$ 39 *and* 175 *are co-prime numbers.*

2. ***The H.C.F. of two co-prime numbers is always 1.***
   Thus, H.C.F. of 15 *and* 16 = 1; *H.C.F. of* 27 *and* 64 = 1 and so on

4.9 DIVISION METHOD

Steps :

1. Divide the greater number by the smaller number.
2. By the remainder of division in Step 1, divide the smaller number.
3. By the remainder in Step 2, divide the remainder obtained in Step 1.
4. Continue in the same way till no remainder is left.

The last divisor is the required H.C.F.

Example 6 :

Find H.C.F. of 36 and 60.

Solution : **[Step 1]** 36) 60 (1 [Dividing the bigger number by the smaller one]
36
[Step 2] 24) 36 (1 [Dividing the smaller number by the remainder of step 1]
24
[Step 3] 12) 24 (2 [Continuing in a similar way]
24
×

Since the last divisor is 12, $\therefore$ **H.C.F. = 12** **(Ans.)**

Example 7 :

Find the H.C.F. of 18, 24 and 32.

Solution :

To find the H.C.F. of more than two numbers :
(i) first find the H.C.F. of any two of the given numbers; then
(ii) find the H.C.F. of the third given number and the H.C.F. obtained in (i).

Let us first find the H.C.F. of 18 and 24.

18) 24 (1
18
6) 18 (3
18
×

H.C.F. of 18 and 24 = 6

Since the third number is 32, and the H.C.F. obtained above is 6, find the H.C.F. of 32 and 6.

```
6 ) 32 ( 5
    30
    ---
     2 ) 6 ( 3
         6
         --
         ×
```

The H.C.F. of 32 and 6 is 2.

$\therefore$ **H.C.F.** of given numbers 18, 24 and 32 = **2** **(Ans.)**

Similarly, in order to find the H.C.F. of four numbers :

1. First of all find the H.C.F. of any three of the given four numbers.
2. Then find the H.C.F. of the fourth number and the H.C.F. obtained in Step 1.

EXERCISE 4(B)

1. Using the ***common factor method***, find the H.C.F. of :
   (i) 16 and 35 (ii) 25 and 20 (iii) 27 and 75
   (iv) 8, 12 and 18 (v) 24, 36, 45 and 60
2. Using the ***prime factor method***, find the H.C.F. the following :
   (i) 5 and 8 (ii) 24 and 49 (iii) 40, 60 and 80
   (iv) 48, 84 and 88 (v) 12, 16 and 28
3. Using the ***division method***, find the H.C.F. of the following :
   (i) 16 and 24 (ii) 18 and 30 (iii) 7, 14 and 24
   (iv) 70, 80, 120 and 150 (v) 32, 56 and 46
4. Use a method of your own choice to find the H.C.F. of :
   (i) 45, 75 and 135 (ii) 48, 36 and 96 (iii) 66, 33 and 132
   (iv) 24, 36, 60 and 132 (v) 30, 60, 90 and 105
5. Find the greatest number that divides each of 180, 216 and 315 completely.
6. Show that 45 and 56 are co-prime numbers.
7. Out of 15, 16, 21 and 28, find out all the pairs of co-prime numbers.
8. Find the greatest number that will divide 93, 111 and 129, leaving remainder 3 in each case.

> Since, 93 – 3 = 90, 111 – 3 = 108 and 129 – 3 = 126.
> $\therefore$ Required number is H.C.F. of 90, 108 and 126.

4.10 MULTIPLES

Since $5 \times 1 = 5$, $5 \times 2 = 10$, $5 \times 3 = 15$
$5 \times 4 = 20$, $5 \times 5 = 25$, etc.

therefore, 5, 10, 15, 20, 25, etc. are multiples of 5.

i.e. Multiples of 5 = 5, 10, 15, 20, 25, ...

i.e. M_5 = 5, 10, 15, 20, 25, 30, ...

In the same way :

(i) Multiples of 3 = 3, 6, 9, 12, 15, 18, ...

i.e. M_3 = 3, 6, 9, 12, 15, 18, ...

$3 \times 1 = 3$, $3 \times 2 = 6$,
$3 \times 3 = 9$, $3 \times 4 = 12$,
$3 \times 5 = 15$ and so on

(ii) M_7 = Multiples of 7 = 7, 14, 21, 28, 35, ...

(iii) M_8 = Multiples of 8 = 8, 16, 24, 32, 40, ...

4.11 LOWEST COMMON MULTIPLE

L.C.M. stands for **Lowest Common Multiple**. The L.C.M. of two or more given numbers is the lowest (smallest) number that is a multiple of each of the given numbers. Thus, ***it is the smallest number which is exactly divisible by each of the given numbers.***

For example :

Multiples of 15 = 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, ...

Multiples of 25 = 25, 50, 75, 100, 125, 150, 175, ...

Common multiples of 15 and 25 = 75, 150, ...

Lowest common multiple = 75

∴ **L.C.M.** of 15 and 25 = **75**

If checked carefully, you will find that 75 is the smallest number which is exactly divisible by both 15 and 25.

4.12 METHODS OF FINDING L.C.M.

The following three methods are most commonly used to find the L.C.M. :

1. *Common Multiple Method* 2. *Prime Factor Method* 3. *Common Division Method*

4.13 COMMON MULTIPLE METHOD

Steps :

1. Find a few multiples of each given number.
2. From the multiples obtained in Step 1, select the common ones.
3. The lowest number (multiple) obtained in Step 2 is the required **lowest common multiple** (L.C.M.) of the given numbers.

Example 8 :

Using common multiple method, find the L.C.M. of :

(i) 4, 5 and 10 (ii) 12, 15 and 20

Solution :

(i) M_4 = Multiples of 4

= 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, ...

M_5 = Multiples of 5

= 5, 10, 15, 20, 25, 30, 35, 40, 45, ...

and, M_{10} = Multiples of 10

= 10, 20, 30, 40, 50, 60, ...

⇒ Common multiples of 4, 5 and 10 = 20, 40, ...

⇒ **L.C.M.** of 4, 5 and 10 = **20**

20 is the smallest number divisible by each of 4, 5 and 10.

(Ans.)

(ii) $\because$ M_{12} = 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, ...

M_{15} = 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, ...

And, M_{20} = 20, 40, 60, 80, 100, 120, 140, ...

$\Rightarrow$ Common multiples of 12, 15 and 20 = 60, 120, ...

$\Rightarrow$ **Required L.C.M. = 60** **(Ans.)**

4.14 PRIME FACTOR METHOD

Example 9 :

Use prime factor method to find L.C.M. of 18, 24 and 36.

Solution :

Step 1 :

Express each of the given numbers as a product of its prime factors and then in index form.

Clearly, 18 = $2 \times 3 \times 3$

= $2^1 \times 3^2$ [Index form]

24 = $2 \times 2 \times 2 \times 3$

= $2^3 \times 3^1$ [Index form]

and, 36 = $2 \times 2 \times 3 \times 3$

= $2^2 \times 3^2$ [Index form]

Step 2 :

L.C.M. = Product of all the prime factors obtained with highest power of each.

Since, the prime factors 2 and 3, obtained above, with highest power are 2^3 and 3^2 respectively.

$\therefore$ **Required L.C.M.** = $2^3 \times 3^2$

= $2 \times 2 \times 2 \times 3 \times 3$ = **72** **(Ans.)**

4.15 COMMON DIVISION METHOD

Example 10 :

Find the L.C.M. of 16, 20 and 24.

Solution :

Steps :

1. Write all the given numbers in a horizontal line, separating them by commas.

$$\begin{array}{l|l} & 16,\ 20,\ 24 \\ \hline \end{array}$$

2. Divide by a suitable number, that exactly divides at least two of the given numbers. And, write down the quotients and the undivided numbers obtained, below the first line.

$$\begin{array}{r|l} 2 & 16,\ 20,\ 24 \\ \hline & \ 8,\ 10,\ 12 \end{array}$$

2	16, 20, 24
2	8, 10, 12
2	4, 5, 6
	2, 5, 3

3. Repeat the process until we get a line of numbers that are prime to one-another.
4. The product of all the divisors and the numbers obtained in the last line will be the required L.C.M.

$\therefore$ **Required L.C.M.** = $2 \times 2 \times 2 \times 2 \times 5 \times 3 =$ **240** **(Ans.)**

Example 11 :

Find the smallest number which, when divided by 8, 12, 16, 24 and 36, leaves no remainder.

Solution :

The smallest number that is exactly divisible by each of the given numbers is their L.C.M.

$\therefore$ Required number = L.C.M. of 8, 12, 16, 24 and 36.

Steps 1, 2 and 3 :

2	8, 12, 16, 24, 36
2	4, 6, 8, 12, 18
2	2, 3, 4, 6, 9,
3	1, 3, 2, 3, 9
	1, 1, 2, 1, 3

Steps 4 :

L.C.M. $= 2 \times 2 \times 2 \times 3 \times 2 \times 3$

$= 144$

$\therefore$ **Req. no. = 144** **(Ans.)**

Example 12 :

Find the smallest number which, when :

(i) decreased by 1 (ii) increased by 3

is exactly divisible by the numbers 21, 45, 63, 81 and 210.

Solution :

First find the L.C.M. of the given numbers.

$\therefore$

3	21, 45, 63, 81, 210
3	7, 15, 21, 27, 70
5	7, 5, 7, 9, 70
7	7, 1, 7, 9, 14
	1, 1, 1, 9, 2

5670 is exactly divisible by each of 21, 45, 63, 81 and 210

$\therefore$ L.C.M. $= 3 \times 3 \times 5 \times 7 \times 9 \times 2 = 5670$

(i) **The required number** = 5670 + 1 = **5671** **(Ans.)**

[**Reason** : On decreasing 5671 by 1, we get 5671 − 1 = 5670; which is the L.C.M. of the given numbers, *i.e.* exactly divisible by each of them]

(ii) **The required number** = 5670 − 3 = **5667** **(Ans.)**

An Important Result : *For any two numbers :*

The product of their L.C.M. and H.C.F. = The product of the numbers.

Consider the numbers 48 and 60.

Their H.C.F. = 12 and their L.C.M. = 240

$\therefore$ The **product of** their **H.C.F. and L.C.M.** = 12 × 240 = **2880**

Also, the **product of** the **given numbers** = 48 × 60 = **2880**

$\therefore$ H.C.F. × L.C.M. of any two numbers = Product of the two numbers.

$\Rightarrow$ (i) L.C.M. of two numbers $= \dfrac{\text{Their product}}{\text{Their H.C.F.}}$

(ii) H.C.F. of two numbers $= \dfrac{\text{Their product}}{\text{Their L.C.M.}}$

and (iii) $\dfrac{\text{Product of L.C.M. and H.C.F.}}{\text{One number}}$ = The other number

EXERCISE 4(C)

1. Using the ***common multiple method***, find the L.C.M. of the following :
   (i) 8, 12, and 24 (ii) 10, 15 and 20 (iii) 3, 6, 9 and 12
2. Find the L.C.M. of each of the following groups of numbers, using (i) **the *prime factor method*** and (ii) **the *common division method* :**
   (i) 18, 24 and 96 (ii) 100, 150 and 200 (iii) 14, 21 and 98
   (iv) 22, 121 and 33 (v) 34, 85 and 51
3. The H.C.F. and the L.C.M. of two numbers are 50 and 300 respectively. If one of the numbers is 150, find the other one.
4. The product of two numbers is 432, and their L.C.M. is 72. Find their H.C.F.
5. The product of two numbers is 19,200, and their H.C.F. is 40. Find their L.C.M.
6. Find the smallest number which, when divided by 12, 15, 18, 24 and 36 leaves no remainder.
7. Find the smallest number which, when increased by one is exactly divisible by 12, 18, 24, 32 and 40.
8. Find the smallest number which, on being decreased by 3, is completely divisible by 18, 36, 32 and 27.

Revision Exercise (Chapter 4)

1. Find the H.C.F. of : (i) 108, 288 and 420 (ii) 36, 54 and 138
2. Find the L.C.M. of : (i) 72, 80 and 252 (ii) 48, 66 and 120
3. State **true** or **false** (Give an example in support of your answer in each case) :
   (i) H.C.F. of two prime numbers is 1.
   (ii) H.C.F. of two co-prime numbers is 1.
   (iii) L.C.M. of two prime numbers is equal to their product.
   (iv) L.C.M. of two co-prime numbers is equal to their product.
4. The product of two numbers is 12096, and their H.C.F. is 36. Find their L.C.M.
5. The product of the H.C.F. and the L.C.M. of two numbers is 1152. If one number is 48, find the other one.
6. (i) Find the smallest number that is completely divisible by 28 and 42.
   (ii) Find the largest number that can divide 28 and 42 completely.
7. Find the L.C.M. of 140 and 168 and is use the L.C.M. obtained to find the H.C.F. of the given numbers.
8. Find the H.C.F. of 108 and 450 and use the H.C.F. obtained to find the L.C.M. of the given numbers.